

# On a moduli space of the Wigner quasiprobability distributions

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Nowadays, due to the quantum engineering needs, a long-standing problem of finding “quantum analogues” for the statistical distributions of classical systems became an actual again. Mathematically this issue can be formulated as a problem of finding the mapping between operators on the Hilbert space of a finite-dimensional quantum system and the Wigner quasiprobability distributions [1] defined over the symplectic flag manifold [2, 3]. Based on the postulates known as the Weyl-Stratonovich correspondence [4, 5], the Wigner quasiprobability distribution for a generic  $N$ -level quantum system can be constructed from two objects: the density matrix  $\rho$  describing a quantum state, and the so-called Stratonovich-Weyl kernel  $\Delta(\Omega_N)$  defined over the symplectic manifold  $\Omega_N$ . According to our recent study [3], the eigenvalues of the kernel  $\Delta(\Omega_N)$  satisfy the set of algebraic equations. In present report an ambiguity in the solution to those “master equations” will be analyzed and the corresponding moduli space of the Wigner quasiprobability distribution will be determined. The general consideration will be exemplified by a detailed description of the Wigner quasiprobability distributions of 2, 3 and 4-dimensional systems.

## References

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