Schutzenberger transformation on graded graphs: Implementation and numerical experiments.

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Overview

Introduction

- 2 Robinson-Shensted-Knuth algorithm
- 3 Plancherel measure
- 4 Schutzenberger transformation
- 5 The connection between RSK and Schutzenberger transformations
- 6 The connection between Plancherel measure and Schutzenberger transformation
- 7 Three-dimensional case
- B Randomization





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Robinson-Shensted-Knuth algorithm

Input: uniformly random permutation of integers from 1 to *n*:

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

Output: a pair of Young tableaux of the same shape:











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Schutzenberger transformation

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9	18						6	13					
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The tableaux obtained by RSK algorithm have a Plancherel distribution. It is a **central measure** on Young tableaux, i.e. the paths between a fixed pair of diagrams have the same probabilities. The probability of a single path to a diagram λ :

$$P_{path}(\lambda_n) = rac{dim(\lambda_n)}{n!}$$

The probability of a diagram λ :

$$P_{diag}(\lambda_n) = rac{dim^2(\lambda_n)}{n!}$$

Schutzenberger transformation (Jeu de taquin)



[Vershik, Kerov'86]: Schutzenberger transformation is applicable for infinite Young tableaux.

11					11				
7					7				
6	9				6	9			
2	4	10	12		2	4	10	12	
1	3	5	8	13	1	3	5	8	13

Initial tableau

Initial tableau

11					11				
7					7				
6	9				6	9			
2	4	10	12		2	4	10	12	
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Initial tableau

Remove the box (0,0)

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Initial tableau

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Initial tableau

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Initial tableau

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Initial tableau

New tableau

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Initial tableau

New tableau

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Initial tableau

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Schutzenberger transformation

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Initial tableau

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Initial tableau

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Initial tableau

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Initial tableau

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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Initial tableau

V. Duzhin, N. Vassiliev (ETU, PDMI)

Schutzenberger transformation

April 20, 2018

Initial tableau

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

April 20, 2018

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

April 20, 2018

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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V. Duzhin, N. Vassiliev (ETU, PDMI)

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15	19	27	36	42	69	72	75				_		14	18	26	35	41	68	71	74				_	
9	14	20	30	39	48	67	73	88	90	97]		8	13	19	29	38	47	66	72	87	89	99		
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2	4	6	10	16	17	33	38	44	54	68	74	92	3	5	9	15	16	30	32	37	43	53	67	73	91
1	3	5	7	11	12	22	32	34	37	50	57	58	1	2	4	6	10	11	21	31	33	36	49	56	57

Initial tableau

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New tableau

Schutzenberger paths



The connection between RSK and Schutzenberger transformations

[Romik, Śniady'15]: RSK gives an isomorphism between the Schutzenberger transformation and the one-sided shift.



There are no known 3D analogue of RSK correspondence.

- How to find a three-dimensional analogue of RSK?
- How to build a central process on 3D Young graph?
- How to calculate the dimension of a 3D Young diagram?



Kerov'93: The coordinates of added boxes in the Plancherel process are distributed according to the semicircle distribution

$$\frac{\sqrt{4-u^2}}{2\cdot\pi}$$

Romik, Sniady'15: The coordinates of Schutzenberger path ends are distributed according to the semicircle distribution.

The distribution of coordinates of boxes (Plancherel, Schutzenberger)

The goal: to compare the distribution of coordinates of boxes added in Plancherel process and the coordinates of last boxes in Schutzenberger paths.

Plancherel process

- Generate a random Plancherel Young diagram of size 3 · 10⁶;
- Build a random Plancherel path from this diagram to the diagram of size $6 \cdot 10^6$;
- On each step save the coordinates of added boxes.

Schutzenberger transformation

- Generate a random Plancherel Young tableau of $3 \cdot 10^6$ boxes;
- Consequently apply the Schutzenberger transformation to tableaux;
- Build the distribution of coordinates on the diagram's front.

The comparison of distribution of coordinates of boxes (Plancherel, Schutzenberger)



The transition probability of the central Plancherel process (2D):

$$p(\lambda \nearrow \lambda') = p(\lambda, x, y) = \prod_{i=0}^{x-1} \frac{h(\lambda, i, y)}{h(\lambda, i, y) + 1} \prod_{j=0}^{y-1} \frac{h(\lambda, x, j)}{h(\lambda, x, j) + 1},$$

where $h(\lambda, x, y)$ is a hook length of an added box (x, y) in a 2D Young diagram λ .



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3D Pseudo-Plancherel random process

The transition probability of the pseudo-Plancherel process (3D):

$$w(\lambda_1 \nearrow \lambda_2) =$$

$$w(\lambda, x, y, z) = \prod_{i=0}^{x-1} \frac{h(\lambda, i, y, z)}{h(\lambda, i, y, z) + 1} \prod_{j=0}^{y-1} \frac{h(\lambda, x, j, z)}{h(\lambda, x, j, z) + 1} \prod_{k=0}^{z-1} \frac{h(\lambda, x, y, k)}{h(\lambda, x, y, k) + 1},$$

where *h* is a hook length of an added box (x, y, z) in a 3D Young diagram λ .

$$p(\lambda_1 \nearrow \lambda_2) = \frac{w(\lambda_1 \nearrow \lambda_2)}{\sum\limits_{\lambda \in V(\lambda)} w(\lambda_1 \nearrow \lambda)},$$

where $V(\lambda)$ is a set of all diagrams which can be obtained by adding a box to λ_1 .

3D Schutzenberger paths ends



3D Schutzenberger paths ends: a histogram



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Schutzenberger transformation

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Cycle lengths for Schutzenberger transformation



tableaux.

A path to a diagram on the third level of Young graph is being selected randomly:



A path to a diagram on the third level of Young graph is being selected randomly:



Randomization

We consequently apply the Schutzenberger transformation on tableaux of the shape $(n{=}10)$



The histogram of paths to this 3D diagram:



Thanks for your attention!