Schutzenberger transformation on graded graphs: Implementation and numerical experiments.

Vasilii Duzhin[1], Nikolay Vassiliev[2]

[1]Saint Petersburg Electrotechnical University "LETI"

[2]Saint Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences

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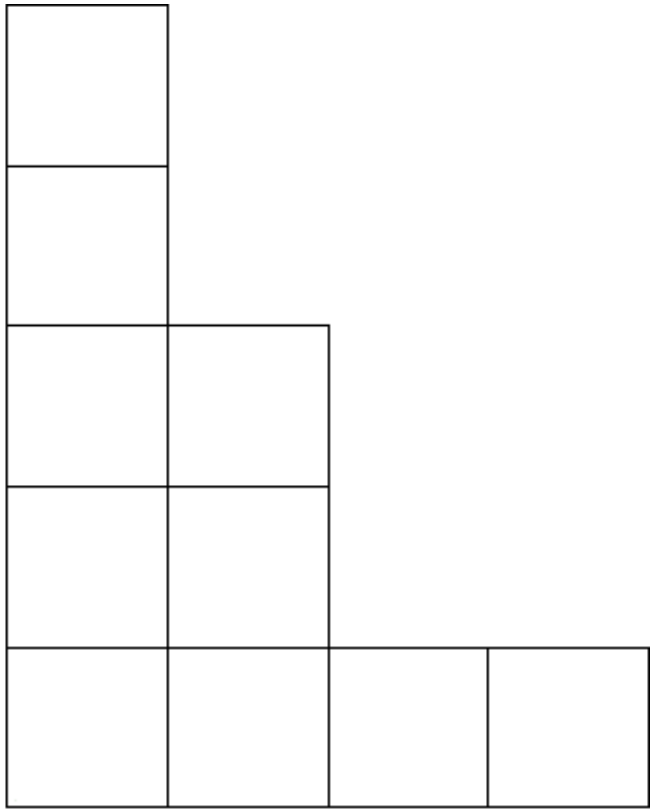
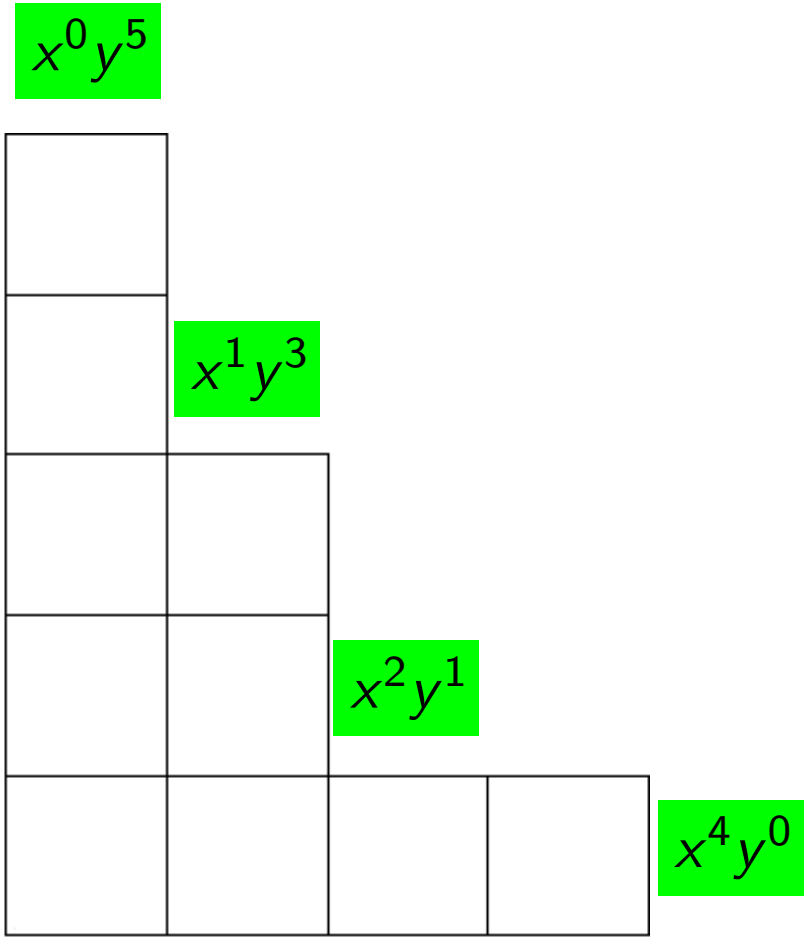
Overview

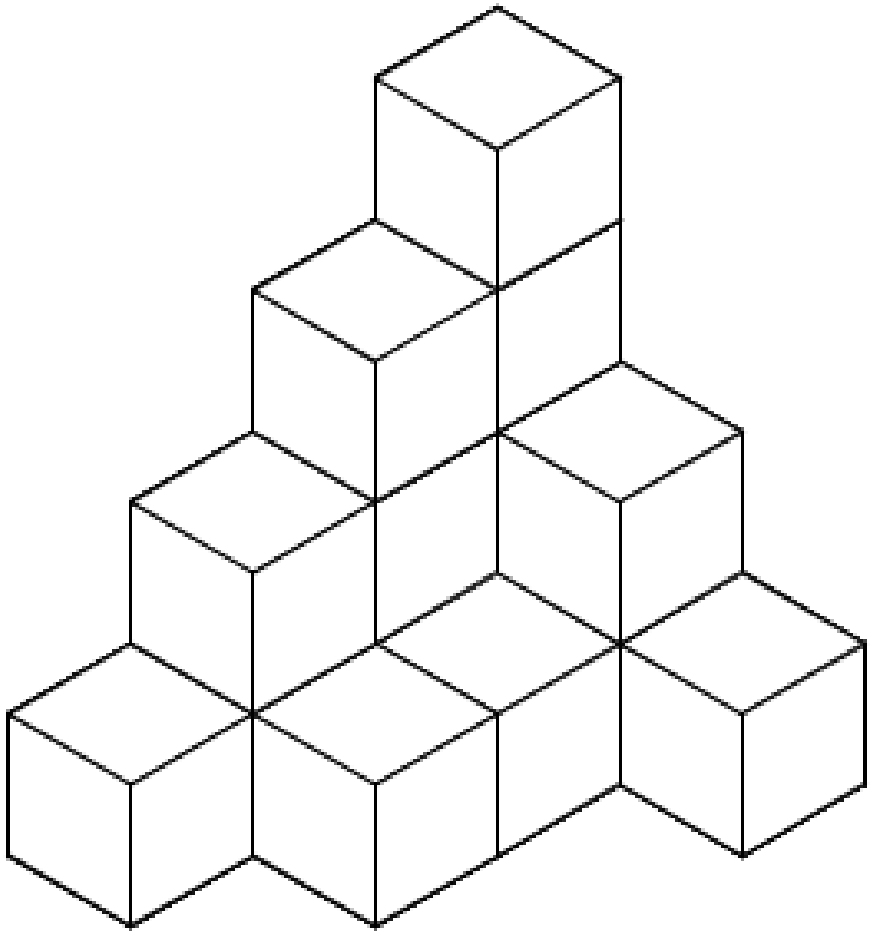
Young diagrams & polynomial ideals



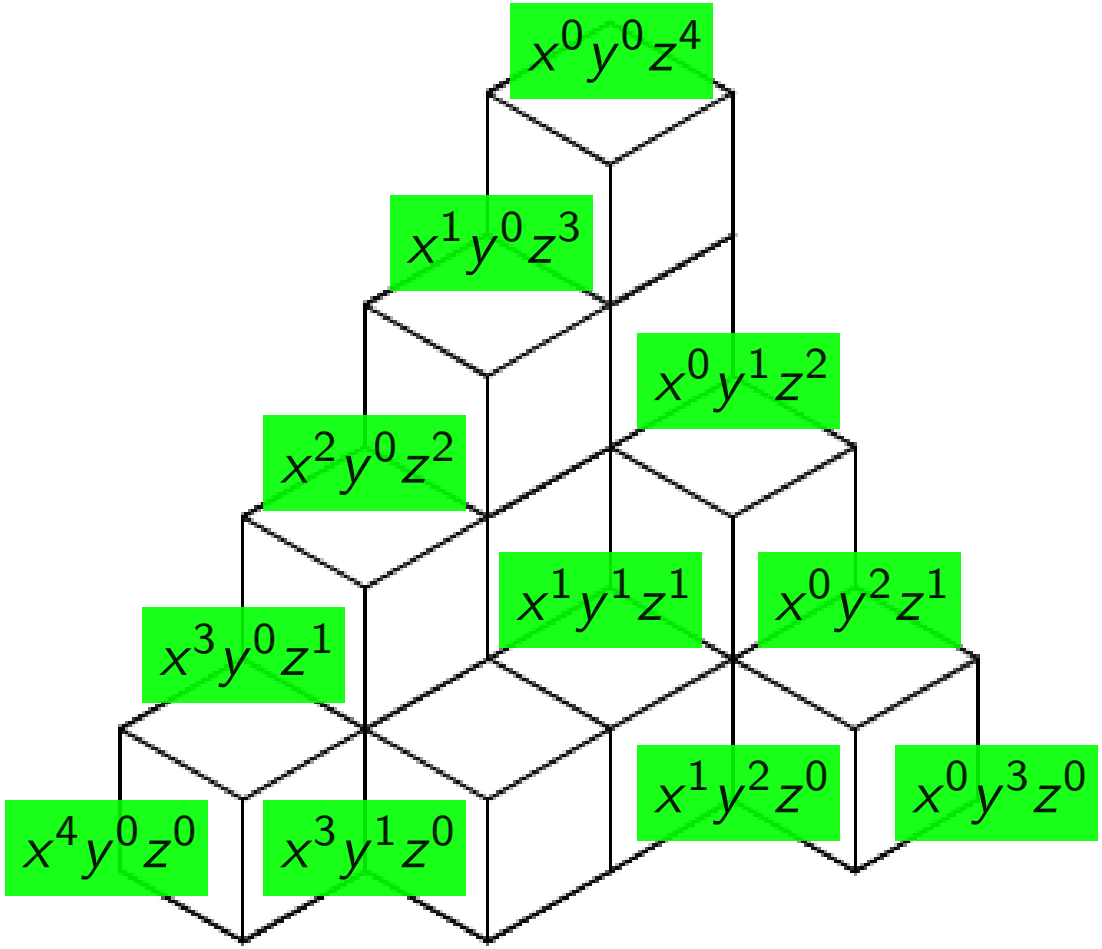
Young diagrams & polynomial ideals



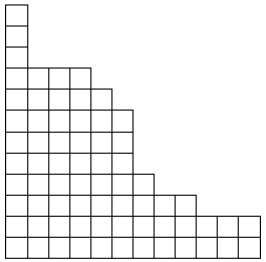
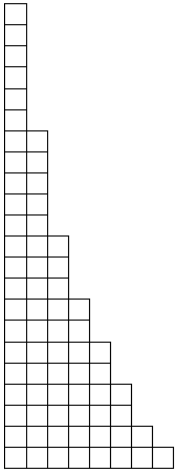
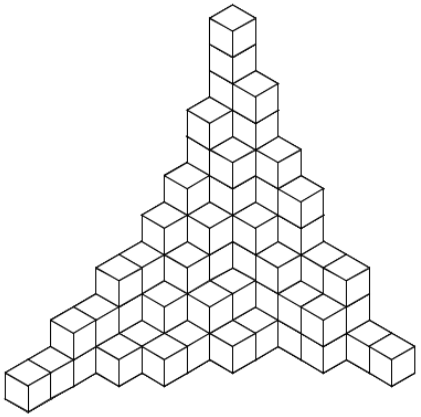
Young diagrams & polynomial ideals



Young diagrams & polynomial ideals



Motivation

		
An **irreducible** representation of symmetric group S(n)	A **projective** representation of symmetric group S(n)	**???**

Robinson-Shensted-Knuth algorithm

Input: uniformly random permutation of integers from 1 to n:

$$13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15$$

Output: a pair of Young tableaux of the same shape:

T_1 :

15				
14				
10	17			
8	9	19		
5	6	18		
3	4	11	16	20
1	2	7	12	13

T_2 :

16				
10				
7	17			
6	13	20		
5	11	14		
3	4	9	18	19
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

13

1

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

2

1

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

2	13

1	2

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

16	
2	13

3	
1	2

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

4	
2	13

3	
1	2

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

4	16
2	13

3	4
1	2

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

7	
4	16
2	13

5	
3	4
1	2

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

9	
7	
4	16
2	13

6	
5	
3	4
1	2

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

12	
9	
7	
4	16
2	13

7	
6	
5	
3	4
1	2

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

12	
9	
7	
4	16
1	13

7	
6	
5	
3	4
1	2

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

12	
9	
7	
4	16
1	2

7	
6	
5	
3	4
1	2

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

12		
9		
7		
4	16	
1	2	13

7		
6		
5		
3	4	
1	2	8

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

12		
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3	16	
1	2	13

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3	4	
1	2	8

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

12		
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3	4	
1	2	8

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

12		
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3	4	9
1	2	8

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

20		
12		
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3	4	16
1	2	13

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5		
3	4	9
1	2	8

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

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3	4	9
1	2	8

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

20		
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7	12	
3	4	16
1	2	13

10		
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5	11	
3	4	9
1	2	8

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

20		
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5	11	
3	4	9
1	2	8

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

20		
11		
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6	7	
3	4	16
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5	11	
3	4	9
1	2	8

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

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3	4	9
1	2	8

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

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3	4	9	
1	2	8	12

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

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5	11		
3	4	9	
1	2	8	12

RSK

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1	2	8	12

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

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1	2	8	12

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

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1	2	8	12

RSK

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5	11	14	
3	4	9	
1	2	8	12

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

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1	2	8	12

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

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RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

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1	2	8	12

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

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RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

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5	11	14		
3	4	9		
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

19				
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9	18			
5	6	20		
3	4	16		
1	2	7	12	13

16				
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7				
6	13			
5	11	14		
3	4	9		
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

17				
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9	18			
5	6	20		
3	4	16		
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6	13			
5	11	14		
3	4	9		
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

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3	4	16		
1	2	7	12	13

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7	17			
6	13			
5	11	14		
3	4	9		
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

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9	18			
5	6	20		
3	4	16		
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5	11	14		
3	4	9		
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

17				
14				
10	19			
9	11			
5	6	20		
3	4	16		
1	2	7	12	13

16				
10				
7	17			
6	13			
5	11	14		
3	4	9		
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

17				
14				
10	19			
9	11			
5	6	18		
3	4	16		
1	2	7	12	13

16				
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7	17			
6	13			
5	11	14		
3	4	9		
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

17				
14				
10	19			
9	11			
5	6	18		
3	4	16	20	
1	2	7	12	13

16				
10				
7	17			
6	13			
5	11	14		
3	4	9	18	
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

17				
14				
10	19			
8	11			
5	6	18		
3	4	16	20	
1	2	7	12	13

16				
10				
7	17			
6	13			
5	11	14		
3	4	9	18	
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

17				
14				
10	19			
8	9			
5	6	18		
3	4	16	20	
1	2	7	12	13

16				
10				
7	17			
6	13			
5	11	14		
3	4	9	18	
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

17				
14				
10	19			
8	9			
5	6	18		
3	4	11	20	
1	2	7	12	13

16				
10				
7	17			
6	13			
5	11	14		
3	4	9	18	
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

17				
14				
10	19			
8	9			
5	6	18		
3	4	11	16	
1	2	7	12	13

16				
10				
7	17			
6	13			
5	11	14		
3	4	9	18	
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

17				
14				
10	19			
8	9			
5	6	18		
3	4	11	16	20
1	2	7	12	13

16				
10				
7	17			
6	13			
5	11	14		
3	4	9	18	19
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

15				
14				
10	19			
8	9			
5	6	18		
3	4	11	16	20
1	2	7	12	13

16				
10				
7	17			
6	13			
5	11	14		
3	4	9	18	19
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

15				
14				
10	17			
8	9			
5	6	18		
3	4	11	16	20
1	2	7	12	13

16				
10				
7	17			
6	13			
5	11	14		
3	4	9	18	19
1	2	8	12	15

RSK

13, 2, 16, 4, 7, 9, 12, 1, 3, 20, 11, 6, 18, 14, 5, 19, 17, 10, 8, 15

15				
14				
10	17			
8	9	19		
5	6	18		
3	4	11	16	20
1	2	7	12	13

16				
10				
7	17			
6	13	20		
5	11	14		
3	4	9	18	19
1	2	8	12	15

Plancherel measure

The tableaux obtained by RSK algorithm have a Plancherel distribution.
It is a **central measure** on Young tableaux, i.e. the paths between a fixed pair of diagrams have the same probabilities.
The probability of a single path to a diagram λ:

$$P_{path}(\lambda_n) = \frac{dim(\lambda_n)}{n!}$$

The probability of a diagram λ:

$$P_{diag}(\lambda_n) = \frac{dim^2(\lambda_n)}{n!}$$

Schutzenberger transformation (Jeu de taquin)

Input: a Young tableau of size n

16				
10				
7	17			
6	13	20		
5	11	14		
3	4	9	18	19
1	2	8	12	15

Output: a Young tableau of size $n-1$

15				
9				
6	16			
5	12			
4	10	19		
2	8	13	17	18
1	3	7	11	14

[Vershik, Kerov'86]: Schutzenberger transformation is applicable for infinite Young tableaux.

Example 1

11				
7				
6	9			
2	4	10	12	
1	3	5	8	13

Initial tableau

11				
7				
6	9			
2	4	10	12	
1	3	5	8	13

Initial tableau

Example 1

11				
7				
6	9			
2	4	10	12	
1	3	5	8	13

Initial tableau

11				
7				
6	9			
2	4	10	12	
	3	5	8	13

Remove the box (0,0)

Example 1

11				
7				
6	9			
2	4	10	12	
1	3	5	8	13

Initial tableau

11				
7				
6	9			
	4	10	12	
2	3	5	8	13

Example 1

11				
7				
6	9			
2	4	10	12	
1	3	5	8	13

Initial tableau

11				
7				
6	9			
4		10	12	
2	3	5	8	13

Example 1

11				
7				
6	9			
2	4	10	12	
1	3	5	8	13

Initial tableau

11				
7				
6				
4	9	10	12	
2	3	5	8	13

Example 1

11				
7				
6	9			
2	4	10	12	
1	3	5	8	13

Initial tableau

11				
7				
6	14			
4	9	10	12	
2	3	5	8	13

Example 1

11				
7				
6	9			
2	4	10	12	
1	3	5	8	13

Initial tableau

10				
6				
5	13			
3	8	9	11	
1	2	4	7	12

New tableau

Example 1

11				
7				
6	9			
2	4	10	12	
1	3	5	8	13

Initial tableau

10				
6				
5	13			
3	8	9	11	
1	2	4	7	12

New tableau

(0,0), (0,1) ,(1,0), (1,1) ,(2,0),(0,2),(0,3),(3,0), (1,2) ,(2,1),(0,4),(3,1),(4,0)

(0,0),(1,0), (0,1) ,(2,0),(0,2),(0,3),(3,0), (1,1) ,(2,1),(0,4),(3,1),(4,0), (1,2)

Example 2

95												
87												
85												
78												
65	97											
64	80											
62	68											
57	61	86	93									
51	55	72	89									
49	53	67	79	91								
27	31	43	47	83	98							
25	30	37	45	63	88							
17	28	29	42	58	81	82	96					
11	21	22	38	44	71	74	77					
10	16	20	32	41	50	69	75	90	92	99		
4	6	15	23	26	33	48	54	66	73	84		
3	5	8	12	18	19	35	40	46	56	70	76	94
1	2	7	9	13	14	24	34	36	39	52	59	60

Initial tableau

95												
87												
85												
78												
65	97											
64	80											
62	68											
57	61	86	93									
51	55	72	89									
49	53	67	79	91								
27	31	43	47	83	98							
25	30	37	45	63	88							
17	28	29	42	58	81	82	96					
11	21	22	38	44	71	74	77					
10	16	20	32	41	50	69	75	90	92	99		
4	6	15	23	26	33	48	54	66	73	84		
3	5	8	12	18	19	35	40	46	56	70	76	94
1	2	7	9	13	14	24	34	36	39	52	59	60

Initial tableau

Example 2

95												
87												
85												
78												
65	97											
64	80											
62	68											
57	61	86	93									
51	55	72	89									
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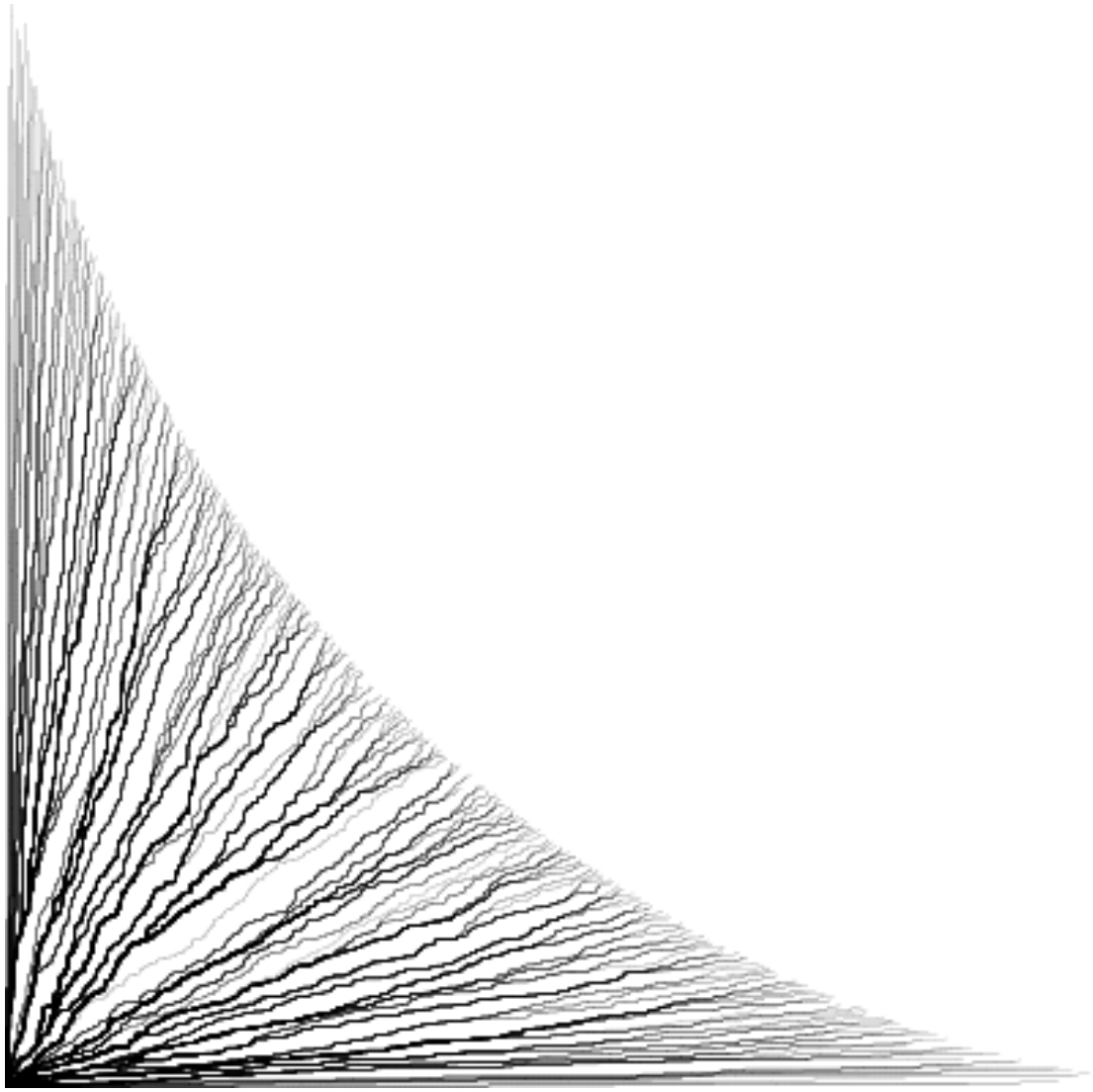
54	58	97	98									
48	52	83	90									
46	50	69	86	88								
28	44	64	76	80	95							
24	27	40	42	60	85							
22	25	34	39	55	78	79	93					
14	18	26	35	41	68	71	74					
8	13	19	29	38	47	66	72	87	89	99		
7	12	17	20	23	45	51	63	70	81	96		
3	5	9	15	16	30	32	37	43	53	67	73	91
1	2	4	6	10	11	21	31	33	36	49	56	57

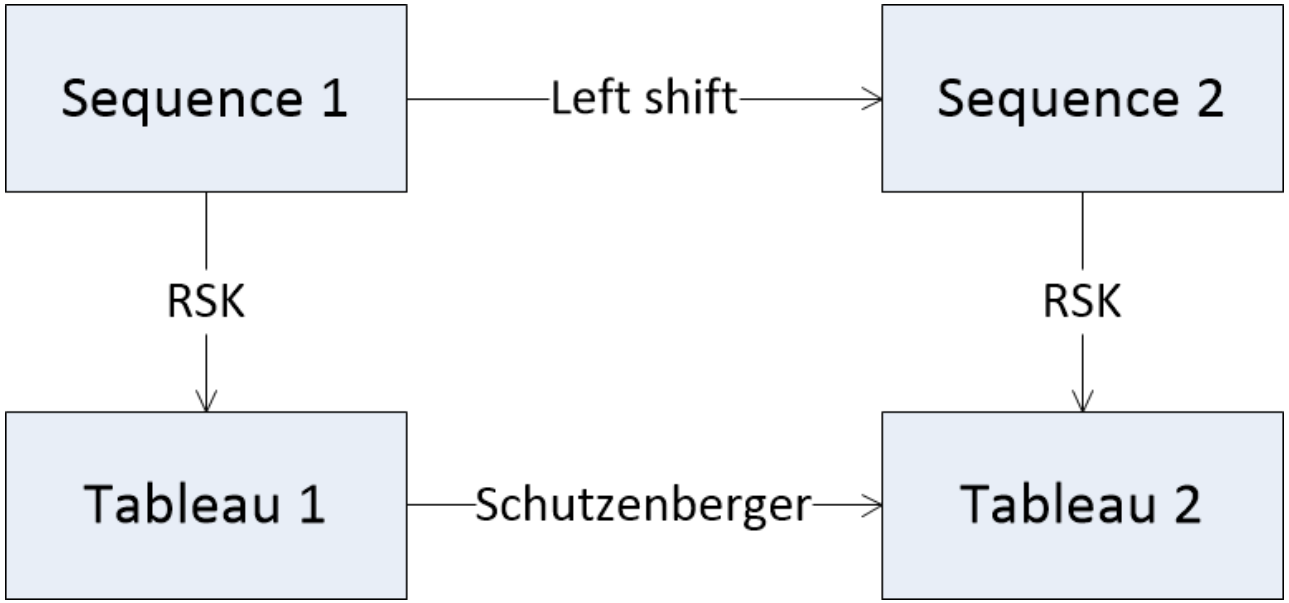
New tableau

Schutzenberger paths



The connection between RSK and Schutzenberger transformations

[*Romik*, *Śniady*'15]: RSK gives an isomorphism between the Schutzenberger transformation and the one-sided shift.

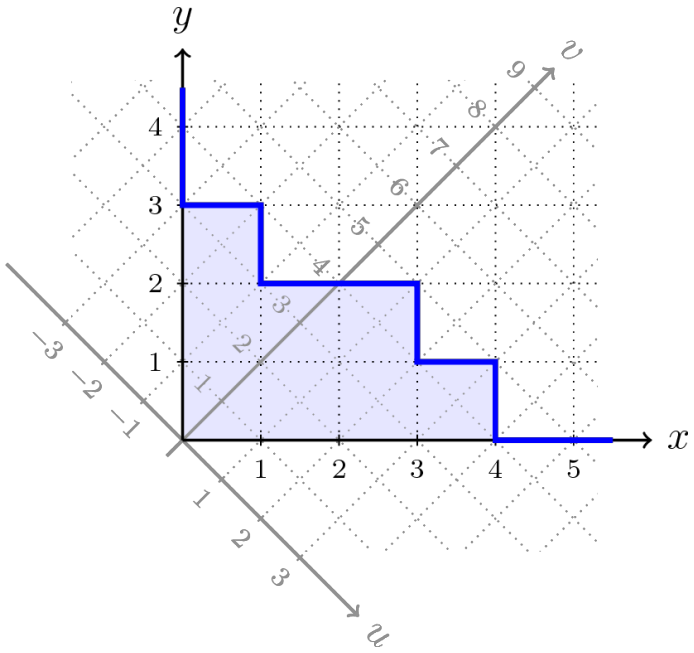
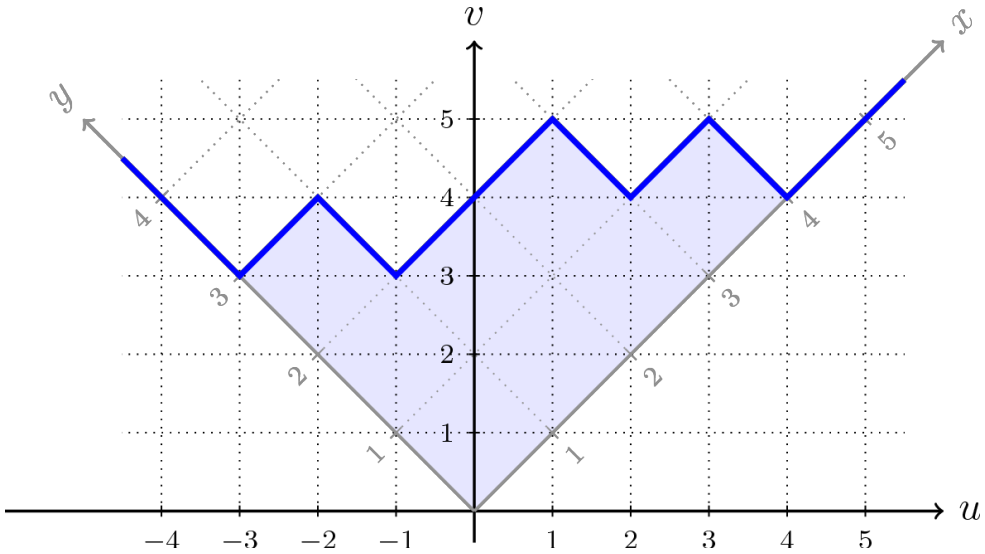


There are no known 3D analogue of RSK correspondence.

Some open problems

- How to find a three-dimensional analogue of RSK?
- How to build a central process on 3D Young graph?
- How to calculate the dimension of a 3D Young diagram?

Different coordinate systems

French notation	Russian notation (Vershik-Kerov coordinates)
	

Plancherel measure and Schutzenberger transformation

Kerov'93: The coordinates of added boxes in the Plancherel process are distributed according to the semicircle distribution

$$\frac{\sqrt{4-u^2}}{2\cdot\pi}$$

Romik, Sniady'15: The coordinates of Schutzenberger path ends are distributed according to the semicircle distribution.

The distribution of coordinates of boxes (Plancherel, Schutzenberger)

The goal: to compare the distribution of coordinates of boxes added in Plancherel process and the coordinates of last boxes in Schutzenberger paths.

Plancherel process

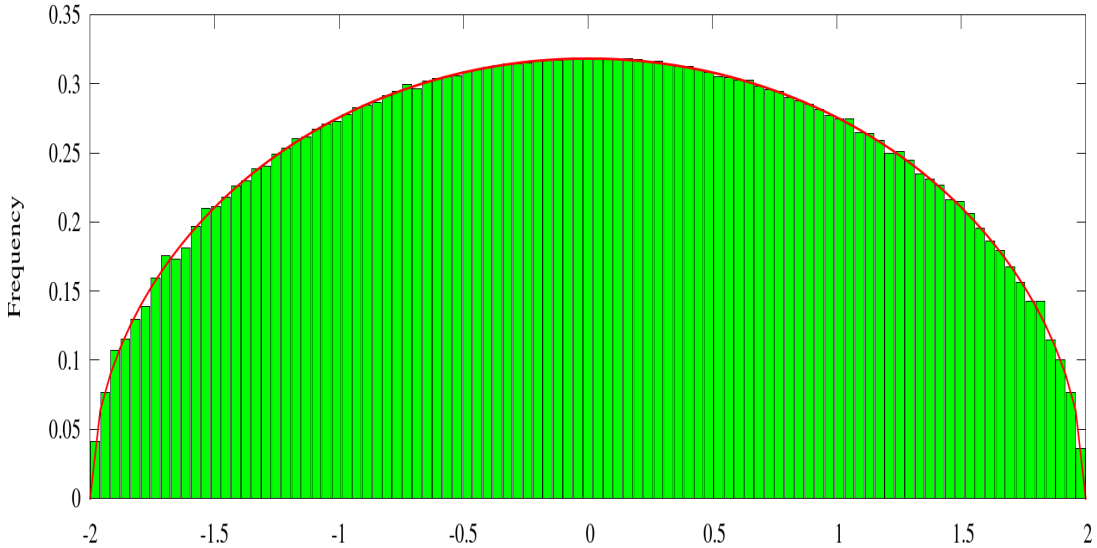
- Generate a random Plancherel Young diagram of size $3 \cdot 10^6$;
- Build a random Plancherel path from this diagram to the diagram of size $6 \cdot 10^6$;
- On each step save the coordinates of added boxes.

Schutzenberger transformation

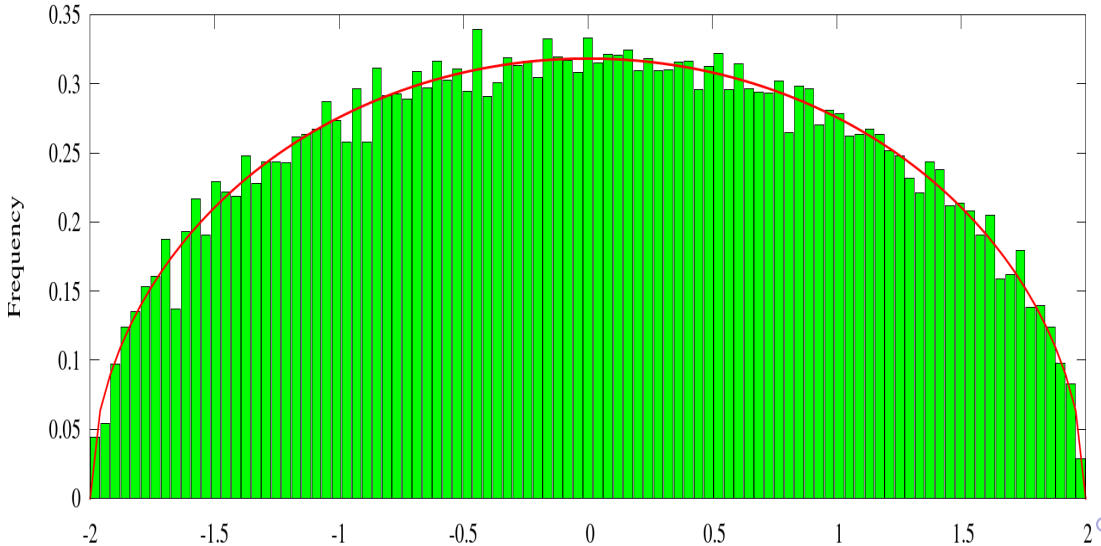
- Generate a random Plancherel Young tableau of $3 \cdot 10^6$ boxes;
- Consequently apply the Schutzenberger transformation to tableaux;
- Build the distribution of coordinates on the diagram's front.

The comparison of distribution of coordinates of boxes (Plancherel, Schutzenberger)

Plancherel coordinates



Coordinates of Schutzenberger paths



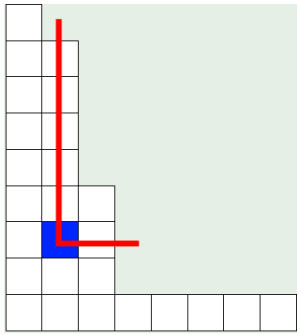
Plancherel random process

The transition probability of the central Plancherel process (2D):

$$p(\lambda \nearrow \lambda') = p(\lambda, x, y) = \prod_{i=0}^{x-1} \frac{h(\lambda, i, y)}{h(\lambda, i, y) + 1} \prod_{j=0}^{y-1} \frac{h(\lambda, x, j)}{h(\lambda, x, j) + 1},$$

where $h(\lambda, x, y)$ is a hook length of an added box (x, y) in a 2D Young diagram λ.

Hook of a box (2,3):



3D Pseudo-Plancherel random process

The transition probability of the pseudo-Plancherel process (3D):

$$w(\lambda_1 \nearrow \lambda_2) =$$

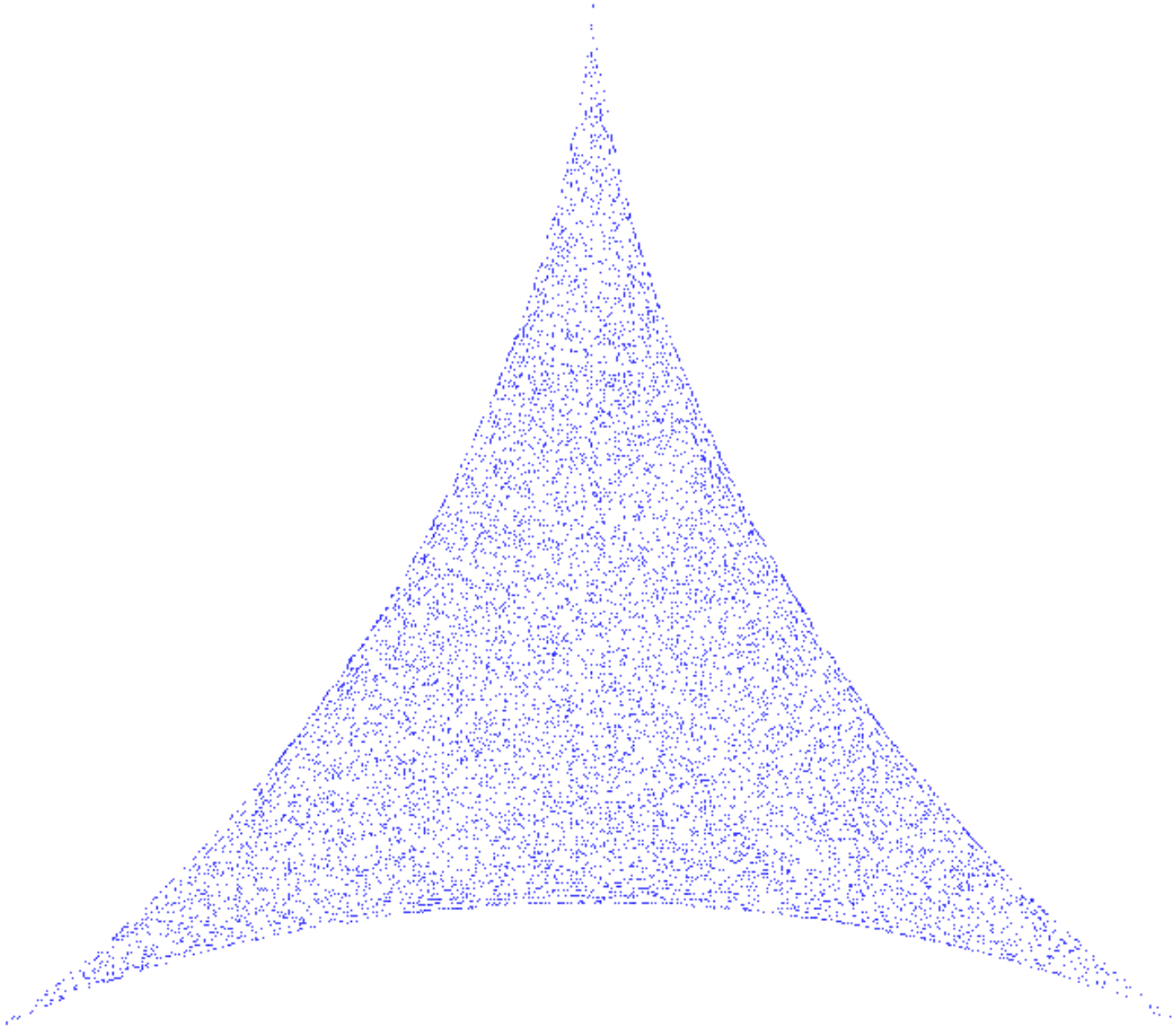
$$w(\lambda, x, y, z) = \prod_{i=0}^{x-1} \frac{h(\lambda, i, y, z)}{h(\lambda, i, y, z) + 1} \prod_{j=0}^{y-1} \frac{h(\lambda, x, j, z)}{h(\lambda, x, j, z) + 1} \prod_{k=0}^{z-1} \frac{h(\lambda, x, y, k)}{h(\lambda, x, y, k) + 1},$$

where h is a hook length of an added box (x, y, z) in a 3D Young diagram λ.

$$p(\lambda_1 \nearrow \lambda_2) = \frac{w(\lambda_1 \nearrow \lambda_2)}{\sum\limits_{\lambda \in V(\lambda)} w(\lambda_1 \nearrow \lambda)},$$

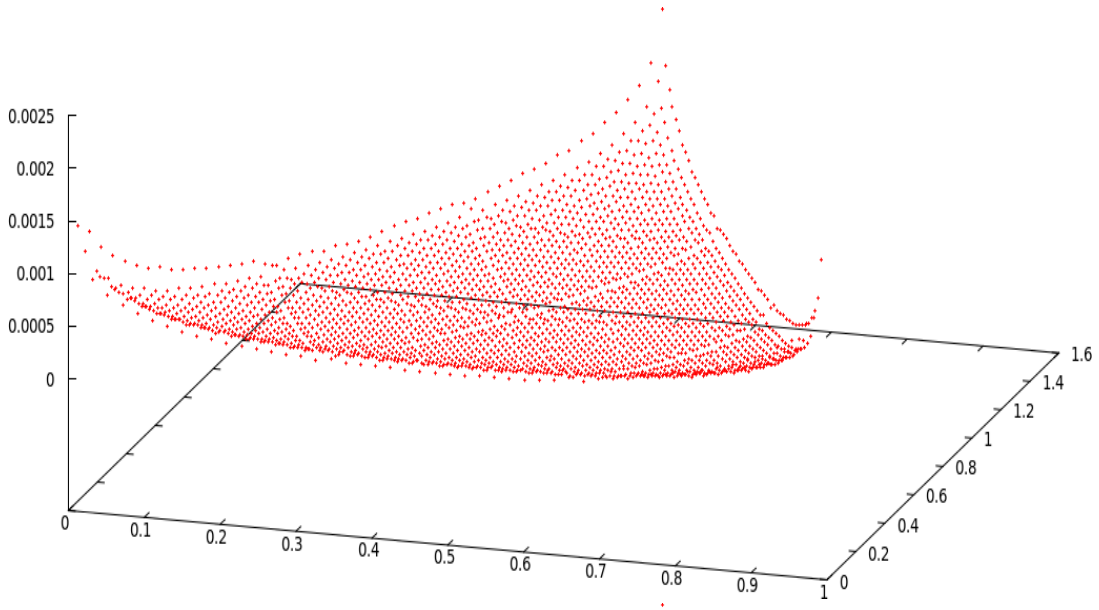
where $V(\lambda)$ is a set of all diagrams which can be obtained by adding a box to λ_1.

3D Schutzenberger paths ends

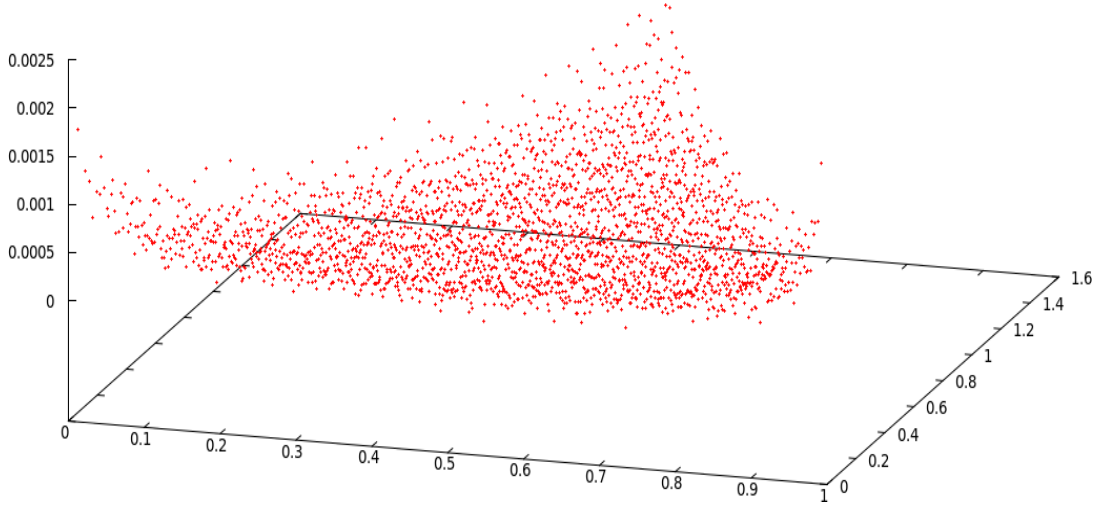


3D Schutzenberger paths ends: a histogram

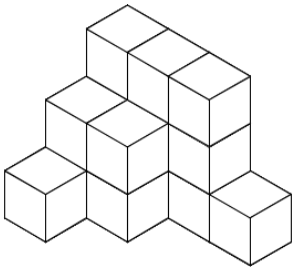
Pseudo-Plancherel coordinates



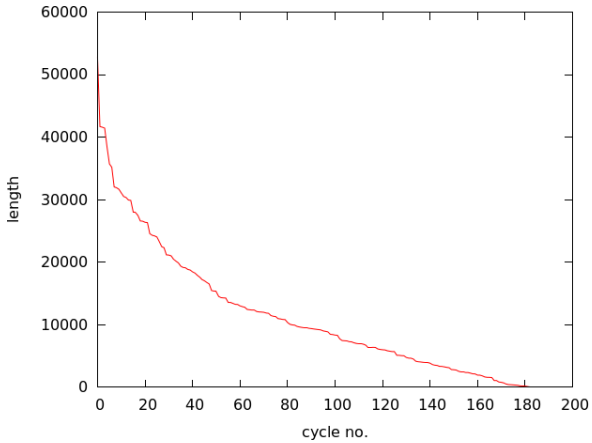
Coordinates of Schutzenberger paths

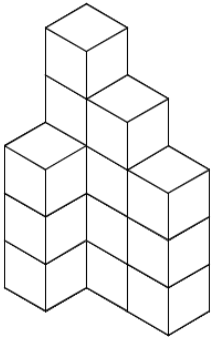


Cycle lengths for Schutzenberger transformation

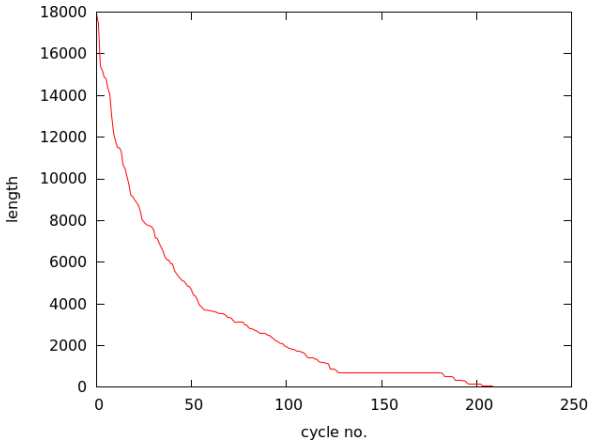


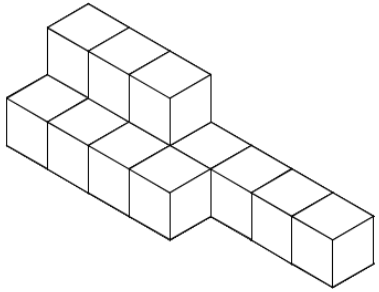
dim=2161967



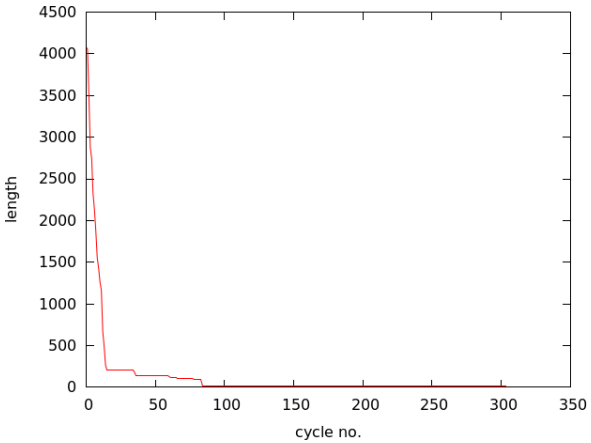


dim=696743





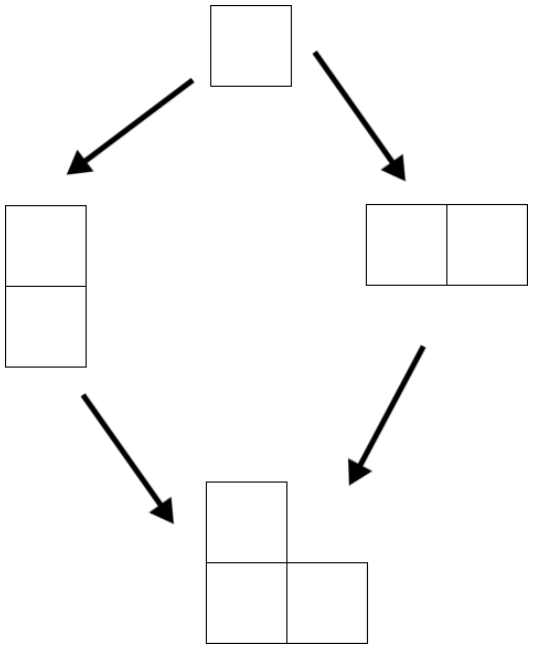
dim=43573



Problem: Schutzenberger transformations do not generate all possible tableaux.

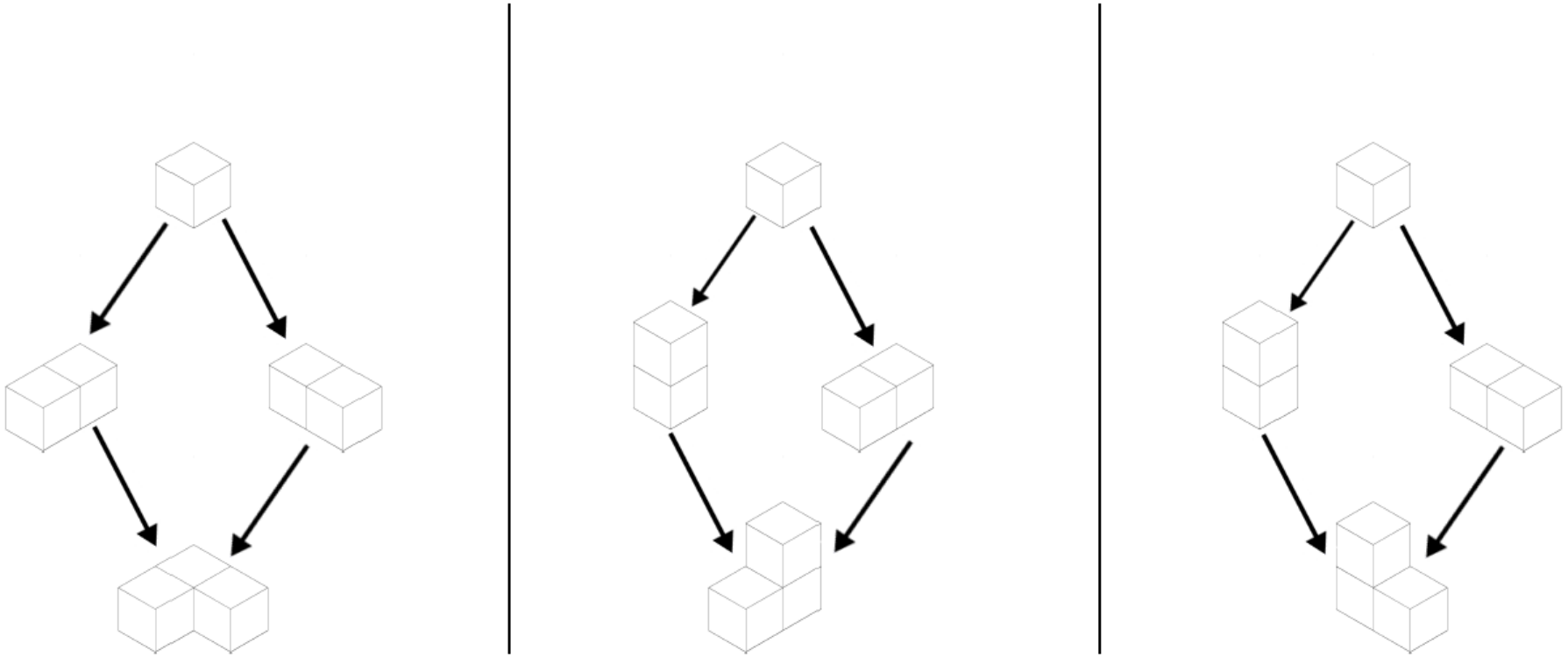
Randomization (2D)

A path to a diagram on the third level of Young graph is being selected randomly:



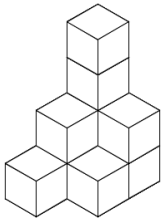
Randomization (3D)

A path to a diagram on the third level of Young graph is being selected randomly:

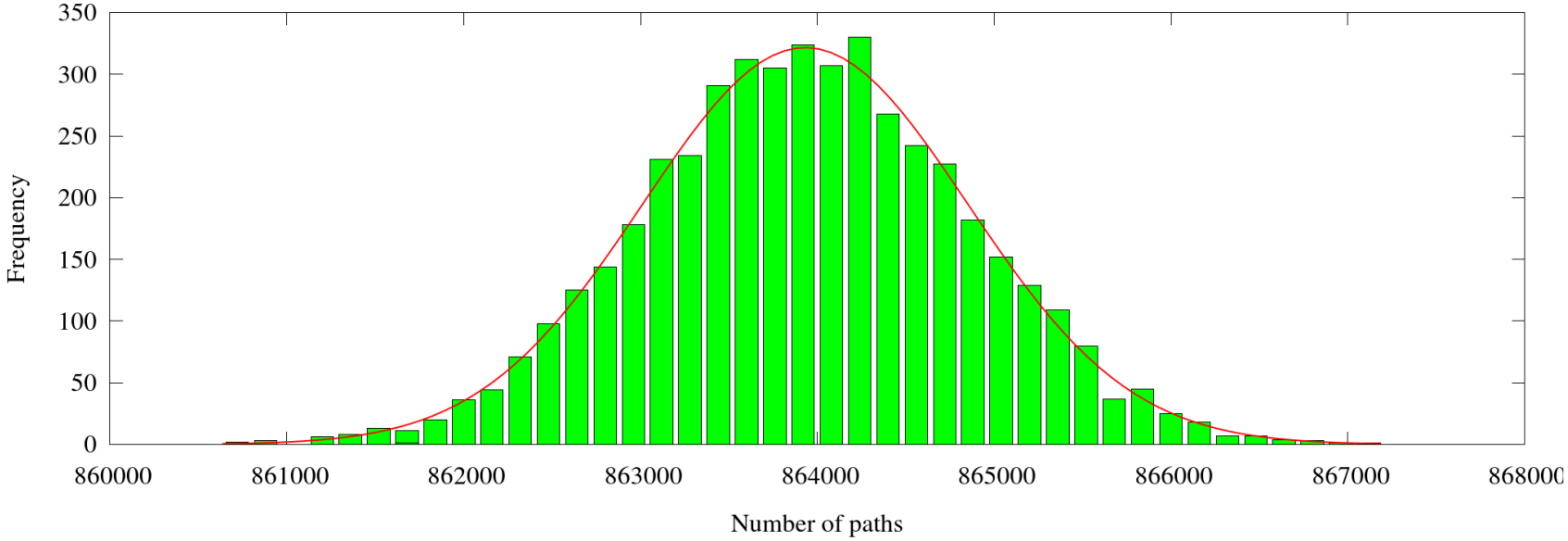


Randomization

We consequently apply the Schutzenberger transformation on tableaux of the shape (n=10)



The histogram of paths to this 3D diagram:



Thanks for your attention!