On complexity of trajectories in the equal-mass free-fall three-body problem

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Abstract. We study complexity of trajectories in the equal-mass free-fall three-body problem. We construct numerically symbolic sequences using different methods: close binary approaches, triple approaches, collinear configurations and other. Different entropy estimates for individual trajectories and for a system as a whole are compared.

We analyse complexity of trajectories in the equal-mass free-fall three-body problem by numerically constructing symbolic sequences and calculating different entropy-like parameters for these sequences. We also discuss some ways to estimate Kolmogorov and Kolmogorov-Sinai entropy.

Symbolic dynamics was used to analyze some special cases of the three-body problem: Alexeyev [2, 3, 4, 5] has found an intermittence of motions of different types in the one special case of the three-body problem - Sitnikov problem. Symbolic dynamics was also applied in two other special cases of the three-body problem: the rectilinear problem (Tanikawa & Mikkola [10, 11]); and the isosceles problem (Zare & Chesley [14, 6]). Tanikawa & Mikkola [12] considered the case with non-zero angular momentum; they also studied free-fall case and have found sequences of triple collision orbits and periodic orbits for isosceles and collinear cases [13].

It is not easy to visualize initial conditions in the general case because of the high dimension of the problem: 3 masses of the bodies + 9 initial coordinates + 9 initial velocities. Equal-mass free-fall three-body problem is much easier and convenient for study: it drastically reduces the dimension of the problem and allows easy visualization of initial configuration. All the masses are equal, so all permutations of the bodies will give us equivalent systems. Since initial velocities are zero, the problem becomes flat, and at any moment of time we have only two components of coordinates and velocities for each body. If at the initial moment we place two bodies in the points $(-0.5; 0)$ and $(0.5; 0)$, then all possible configurations will be covered if we place the third body inside the region D bounded by two

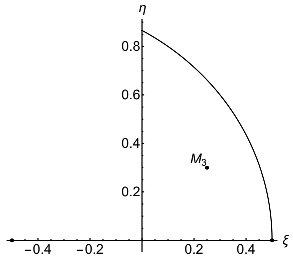

FIGURE 1. Agekian-Anosova region D.

straight line segments and arc of the unit circle centered at $(-0.5, 0)$ (Fig. 1) [1]. All other possible initial configurations (with zero initial velocities) can be received by the projection to this region D and (if needed) transformation of time.

We used code by Seppo Mikkola (Tuorla Observatory, University of Turku) [8] for numerical simulations.

Typical final stage of the evolution of three-body system is close binary moving in one direction, while the third body moves in the opposite direction. So, of interest is finite segment of the symbolic sequences while (infinite) final parts of these sequences are predictable (the only difference is which of the bodies is ejected). If one will calculate entropy of such "infinite" sequence, the result is obvious. So, we study the evolution of the system during the finite period of time (anyway, we can not integrate it infinitely long), considering the stage of active interaction between the bodies. This way, we study complexity of finite sequences and in our "numerical symbolic dynamics" approach we replace original three-body system by a dynamical system that behaves like our original system during this period of time, and have similar behavior all other time (without disruption).

We scan region D with step 0.0005 on each coordinate. For each starting point we numerically integrate equations of motion, construct symbolic sequences and estimate entropy of each sequence. One can use different methods to construct symbolic sequences (see e.g. [9]). In this study, we construct symbolic sequences using binary encounters (we detect minimum distance between two bodies, and corresponding symbol is the number of the distant body, i.e. symbols are from the alphabet $\{1, 2, 3\}$), triple encounters (we detect minimum of the sum of all three mutual distances between bodies, corresponding symbol is the number of the distant body, again symbols are from the alphabet $\{1, 2, 3\}$), using collinear configuration (similar to [13] we detect the moment when one body crosses the line connecting two distant bodies, there are 6 different configurations possible) and projection to the Agekian-Anosova region D (in [7] authors call it homology mapping – there are 6 possible combinations of projecting our three bodies to the region D, thus the alphabet in this case is $\{1, 2, 3, 4, 5, 6\}$).

We also estimate Kolmogorov-Sinai entropy using same approach as in [7] – study spreading of projection of neighboring trajectories on the homology map, but while in [7] authors used a "drop" consisting of 100 initial points, we use only nine points (point under consideration and 8 neighbours around it in our grid of initial conditions).

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