Links between second-order Fuchsian equations and first-order linear Fuchsian systems

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The simplest second-order Fuchsian equation is the hypergeometric equation with three Fuchsian singularities. The next in complexity is the Heun equation with four Fuchsian singularities. In addition to Fuchsian singularities apparent singularities can be added to both discussed equations and we arrive to deformed equations.

On the other hand, linear first-order systems with three or four Fuchsian singularities are studied. Clearly, each component of its solution satisfies second-order equation. The number of apparent singularities differs from zero to two according to degree of off-diagonal matrix element in the system.

The number of oarameters in the system is larger than that in the equation. Hence in order to find a needed relation between the given system and the equation it is needed to simplify the matrices – residues at finite Fuchsian singularities. The most simple way to do it is to choose either determinants or traces of these residues to be zero. Calculations for both cases are presented in the talk. Comparitions of these two approaches gives the possibility to find the relation between use of antiquantization procedure [1, 2] and isomonodromic property [3, 4] for derivation Painlevé equation P^{VI}.

References

[1] Slavyanov S. 2015 Antiquantization and the corresponding symmetries *Theoret. and Math. Phys.* **185** 1522-1526

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[3] Bolibruch A 2009 *Inverse problems of monodromie in analytical thery of differential equations* in russian, (Moscow - MNCMO)

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