

# Belly polynomials in Mathematica and asymptotic solutions of integral equations.

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The importance of classical polynomials is widely recognized in the theory of ordinary linear differential equations. Less known is the role of Bell polynomials in the theory of functional equations both linear and nonlinear. Here a short review of modern state of Belly polynomials implemented in CAS Mathematica is given and also an example of their use at solution of functional equations arising in the theory of integral equations with rapidly oscillating kernels is exposed.

Asymptotic solution of Fredholm equation with a large parameter in the exponent leads to the necessity for solution the following two functional equations [1], [2] one linear and the other nonlinear

$$\frac{\partial F(z, s)}{\partial s} + \frac{\partial F(s, t)}{\partial s} = 0 \quad (1)$$

with  $s = \phi(z)$ ,  $t = \phi(\phi(z))$ .

$$h(z) = \mu h(\phi(z))T(z, \phi(z)) \quad (2)$$

Here  $\phi(z)$  and  $h(z)$  are unknown functions denoted as the phase and the amplitude while  $F(z, \phi)$  and  $T(z, \phi)$  are known functions which are regular in an appropriate vicinity of the origin. The parameter  $\mu$  plays the role of an eigenvalue.

The rigorous mathematical approach to the posed problem is extremely difficult since the small denominators arise. However abstracting from these mathematical heights the approximate solution in terms of polynomials can be constructed near the origin. The actions with polynomials include differentiation of superposition of functions and calculation the inversion of functions. For these purposes handling with Belly polynomials implemented in CAS Mathematica can give a needed help. The proposed approach can be used in various physical problems, for instance, in the theory of open resonators.

**References**

- [1] S. Slavyanov *The construction of the asymptotics of the eigenfunctions of Fredholm type integral equations with symmetric rapidly oscillating kernels.* in: Scattering Theory. Theory of oscillations. Probl. Mat. Fiz. v. 6, pp 134-141. 1973 (in Russian).
- [2] S. Slavyanov, *On the theory of open resonators.* Zh. Exp. Teor. Fiz., Vol. 64, pp 785-795, 1973.

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