RSK bumping trees and a fast RSK algorithm

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The Robinson-Schensted-Knuth (RSK) correspondence is a bijection between a set of permutations of integers and a set of pairs of Young tableaux of the same shape: insertion tableaux P and recording tableaux Q. The procedure of transforming the input permutation into tableaux is also known as the RSK algorithm or the RSK transformation. The RSK algorithm has many important applications in combinatorics and representation theory.

Each number from the input permutation is being put into a certain place of tableau P, consequently displacing other numbers when it is necessary. A *bumping route* [1] is a sequence of positions in insertion tableau where bumping occures during the RSK transformation. At the same time, tableau Q "records" the position where the form of P has changed by putting the index of the current number at the same position. A more detailed description of the RSK algorithm can be found in [2].

The RSK algorithm can be easily generalized to the infinite case: the input infinite sequence of numbers can be transformed into a pair of infinite Young tableaux. Some results of numerical experiments using RSK transformation of extremely large input sequences are shown in [3]. For such experiments, the efficiency of the algorithm becomes especially crucial.

The goal of this work is to implement a special variant of RSK which works significantly faster than the original algorithm. The computational costs of the RSK are mainly caused by searching a position where the next number should be bumped in tableau P. In order to solve this problem, we consider tableau P together with a special combinatorial object called a *bumping forest* which is a union of all possible bumping routes of an insertion tableau. The Figure 1 (a) shows an example of a Young tableau equipped with a bumping forest.

The bumping forest itself is shown in Fig. 1 (b). It can be easily seen that it consists of connected components, which we call *bumping trees*. Each bumping tree is a union of bumping routes converging to a same position where the form of tableaux P,Q can be changed.

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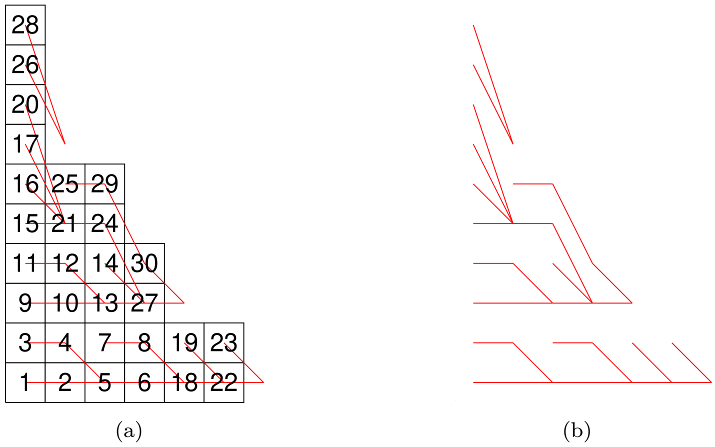

FIGURE 1. A Young tableau and its bumping forest

At each step of the algorithm we need to maintain the correct structure of a bumping forest. This maintenance expenses costs us some computational resources. On the other hand, we do not need to calculate bumping routes for the input numbers. The numerical experiments show that the proposed algorithm works faster than the standard RSK algorithm and the performance gain is higher for larger permutations. Table 1 shows the calculation time of both standard and proposed versions of RSK for different uniformly-distributed random permutations of integers. Each value is an average elapsed time of processing 300 permutations of the same size.

TABLE 1. The comparison between the speed of standard and fast RSK algorithms

Permutation size	Elapsed time by standard RSK (in sec)	Elapsed time by fast RSK (in sec)
100000	5	1
250000	22	2
500000	72	5
1000000	274	15

As we can see from the table, the proposed algorithm works ≈ 18 times faster than the standard one for the permutations of size 10^6.

Note that the fast inverse RSK transformation can be implemented using the bumping forest as well. In that case the bumping routes will be reversed.

References

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