Work distribution between the student and the computer while solving tasks in distant contests.

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Abstract. The work is devoted to distant contests held with the support of interactive environment. While creating a task of this type one always need to choose which activity is left fot the student and which one is left to the computer.

1. Background.

In [?] (based on the role of the tool in the child development [?, ?] and interiorization mechanism [?]) is shown that students' mastering algorithms with mathematical objects goes through several stages. At first, the algorithm is executed consciously, which corresponds to the concept of skill, then the algorithm is convolved, and the student, performing mental actions, actually works with the names of the algorithms, that is, convolutions of the algorithms that are deployed in the process of their application and remain minimized in the process of reasoning built on their basis. The study of the mechanisms of internalization in a computer environment is presented in the works of Papert [?] and his followers.

2. Interactive tasks in distant contests

While holding our contests (Construct, Test, Explore! and the Olympiad in Discrete Mathematics) we use a method of software supported subject tasks [?]. By subject task we mean a task with some understandable real-world statement that does not need any specific knowledge to understand it and to make at least first steps in the solution. By software supported we mean that a task is accompanied with a computer tool, that demonstrates the statement and allows for searching for a solution. Thus, such tasks have to be constructive, and a tool exposes their constructive nature. This tool supporting teh task we call a manipulator.

Within the framework of Olympiad in Discrete Mathematics and Theoretical Informatics we generally use six types of constructive tasks. Each of them is supported by its own manipulator. For CTE contest a new manipulator is developed for any new task.

For example, graph task may be formulated as "Find the minimal graph satisfying the certain conditions". Correctness of the constructed graph is verified automatically. The student can gain additional points for proving the minimality in the text form.

3. Internal and external operations.

While solving research problems, the student is faced with new concepts that, at the time of the occurrence of troubles, may not have representations known by the student. Creating a computer environment for operating with a subject environment allows you to hide (convolve) part of the algorithms that become a part of the environment and are executed automatically. At the same time, other operations are controlled by the student interacting with the environment. Thus, it becomes possible to control the mechanism of interiorization through carrying out the certain actions into the external environment.

4. Example of task. Network repair.

Ten buttons are somehow connected by elastic bands. Each button is removed from this web in turn along with the elastic bands connecting it to the rest of the network. The resulting ten meshes with nine buttons are shown at the top of the screen. Your goal is to reconstruct the original network.

This task is related to reconstruction conjecture which states that every graph with at least three vertices can be reconstucted by the multiset of its vertex-deleted subgraphs (called deck).

The program interface allows not only to clearly demonstrate the solution, but also to examine the deck obtained from its own graph along with the original deck, track the matching subgraphs of the decks, and arrange the elements of the deck in a convenient manner. It provides a meaningful process of experiments, not limited to simple guessing.

In this task the concept of isomorphism plays an auxiliary role, making it possible to quickly verify compliance with the requirements for the constructed solution. The participant has the opportunity to use the idea of isomorphism, moving the vertices of the subgraphs to better understand the reaction of the program. Programs are distinguished by a tool for working with graphs. Thus, the use of the concept of isomorphism in this task without its mathematical definition is compensated by the fact that the participant is given a tool that allows experimentally checking isomorphism, turning one representation of the graph into another.

This possibility of convolution of some skills and «deployment» of others, besides the possibilities of using the mechanism of interiorization, opens up possibilities for a more independent presentation of various ideas of mathematics.

5. Questions

How to properly divide the student's research activities into external operations performed by the student and internal operations performed by the computer?

Should all the algorithms that are convolved be brought outside the human mind?

The classical view on the formation of skills (in other words, the ability to use a set of algorithms that are not explicitly formulated, that is, in a formal algorithmic language) statess that when building a new skill based on the previous ones, the students should first master the previous skills until they are convolved mechanically.

At the same time you can use other methods of accessing basic entities. The latter can be immediately perceived by the learner as a real object that has understandable properties, and then on these entities you can build skills (algorithms) of a higher level, without reducing them to basic ones.

This happens in programming, almost none of the students can imagine the work of the algorithm in terms of electrical signals or even in assembler instructions but they still can use a computer and write programs.

6. Example of task. Steiner tree problem.

The task is about connecting a set of vertices on the plane by edges with possible adding another vertices. The goal is to minmize the total length of edges.

This task could be divided in two parts: the geometrical and the topological or graph one. The geometrical part of the solution is that the minimum is reached when every vertice added is a Torricelli point of a triangle connected with its vertices. The second part of the task has no effective algorithm to solve.

In our version of this task the geometrical part of the solution was performed by the computer: the student should just build the optimal topological configuration. It measn that two topologically equivalent solutions would be evaluated equally.

7. Conclusions

In case of graph tasks the possibility of combining algorithms for working with data in mathematical models is shown so that some operations are displayed externally, that is, they are interface elements and are controlled by the student, while others are presented in a minimized form, that is, these actions are performed automatically by modeling program.

Using various interpretations of mathematical concepts in simulation computer models, it becomes possible to form the skills of working with some algorithms. This makes it possible to more freely organize educational material when operations with complex mathematical concepts can be completed before the components of these concepts are studied in detail.

The use of the convolved algorithms in research problems with computer support allows us not to focus on the insignificant details of the plots and to pose new research problems that form a new mathematical intuition and which could not be posed without using computer support.

The work was supported by the Russian Foundation for Basic Research (Project No 18-013-01130).

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