# Surface electromagnetic waves

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**Abstract.** In the report we discuss surface electromagnetic waves propagating along the boundary of isotropic and anisotropic media. We show how these waves can be investigated in CAS Sage.

In the 1980s, surface waves were discovered that propagate along the interface between two dielectrics without loss [1, 2]. In [3, 4] the first analytical expressions were obtained manually for solutions that are waves propagating along the interface of an anisotropic medium with permittivity

$$\epsilon = \mathrm{diag}(\epsilon_o, \epsilon_o, \epsilon_e).$$

and isotropic medium with constant permittivity $\epsilon$.

For definiteness, let the plane $x = 0$ serve as the interface. The field in the anisotropic medium ($x < 0$) is sought in the form

$$\vec{E} = \left( a_o \vec{E}_o e^{p_o x} + a_e \vec{E}_e e^{p_e x} \right) e^{ik_y y + ik_z z - i\omega t},$$
$$\vec{H} = \left( a_o \vec{H}_o e^{p_o x} + a_e \vec{H}_e e^{p_e x} \right) e^{ik_y y + ik_z z - i\omega t}$$

Here $\omega$ is the circular frequency of the wave, $k_0 = \omega/c$ is the wave number, $\vec{k}_\perp = (0, k_y, k_z)$ is its wave vector, $a_o, a_e$ is the amplitude of two partial waves, and positive numbers $p_o, p_e$ characterize the rate of wave decay in the anisotropic medium. Maxwell's equations give

$$p_o^2 = k_y^2 + k_z^2 - \epsilon_o k_0^2$$
$$p_e^2 = k_y^2 + \frac{\epsilon_e}{\epsilon_o} k_z^2 - \epsilon_e k_0^2$$

and for the vectors $\vec{E}_o, \dots, \vec{H}_e$, explicit expressions are obtained, which we will not present here.

For the isotropic medium ($x > 0$) the field is described by similar formulas

$$\vec{E} = \left( b_o \vec{E}'_o + b_e \vec{E}'_e \right) e^{-px} e^{ik_y y + ik_z z - i\omega t},$$
$$\vec{H} = \left( b_o \vec{H}'_o + b_e \vec{H}'_e \right) e^{-px} e^{ik_y y + ik_z z - i\omega t},$$

but now the constant $p$, which characterizes the decrease in the field in the isotropic medium, turns out to be the same:

$$p^2 = k_y^2 + k_z^2 - \epsilon k_0^2.$$

The conditions for matching electromagnetic fields at the interface lead to a system of homogeneous linear equations for the amplitudes $a_o, a_e, b_o, b_e$. The condition of zero determinant of this system gives the equation

$$\big((k_z^2 - \epsilon k_0^2)p_o + (k_z^2 - \epsilon_o k_0^2)p\big)\big((k_z^2 - \epsilon k_0^2)\epsilon_o p_e + (k_z^2 - \epsilon_o k_0^2)\epsilon p\big) = (\epsilon_o - \epsilon)^2 k_y^2 k_z^2 k_0^2 \tag{1}$$

Thus, we manage to reduce the study of the existence of surface waves to a purely algebraic problem: if in the domain of variation of five variables $k_y k_z p_o p_e p$, in the region specified by the inequalities

$$p_o > 0, \quad p_e > 0, \quad p > 0,$$

the system of algebraic equations

$$\begin{cases} p_o^2 = k_y^2 + k_z^2 - \epsilon_o k_0^2 \\ p_e^2 = k_y^2 + \dfrac{\epsilon_e}{\epsilon_o} k_z^2 - \epsilon_e k_0^2 \\ p^2 = k_y^2 + k_z^2 - \epsilon k_0^2 \end{cases}$$

together with Eq. (1) has a solution, then this solution corresponds to a field satisfying Maxwell's equations, matching conditions at the interface, and exponentially decreasing with distance from the interface.

It is possible to eliminate $k_0$ from this system by assuming

$$p = k_0 q, \quad p_o = k_0 q_o, \quad p_e = k_0 q_e$$

and

$$k_y = k_0 \beta, \quad k_z = k_0 \gamma.$$

Then the system of equations is written in the form

$$\begin{cases} q_o^2 = \beta^2 + \gamma^2 - \epsilon_o \\ q_e^2 = \beta^2 + \dfrac{\epsilon_e}{\epsilon_o}\gamma^2 - \epsilon_e \\ q^2 = \beta^2 + \gamma^2 - \epsilon \\ \big((\gamma^2 - \epsilon)q_o + (\gamma^2 - \epsilon_o)q\big)\big((\gamma^2 - \epsilon)\epsilon_o q_e + (\gamma^2 - \epsilon_o)\epsilon q\big) = (\epsilon_o - \epsilon)^2 \beta^2 \gamma^2. \end{cases} \tag{2}$$

This system defines a curve in the space $\beta\gamma q q_o q_e$ and we are interested to know whether this curve falls within the region

$$q > 0, \quad q_o > 0, \quad q_e > 0.$$

The following observation allowed us to move forward: this curve can be described relatively simply if we consider its projection not on the $\beta, \gamma$ plane, which we tried to do first of all, but on the $q_o q_e$ plane. It turned out that the curve has genus zero and all variables are expressed in radicals through the value

$$t = p_e/p_o.$$

The talk will be devoted to the study of this curve in the Sage computer algebra system.

We believe that the use of computer algebra methods will make it possible to investigate the essentially algebraic question of the existence of surface waves as completely as it deserves due to its obvious applied significance [4].

**Acknowledgments.** This work was supported by the Russian Foundation for Basic Research, projects nos. 16-07-00556 and 18-07-00567.

## References

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