Growth in groups and the number of curves and knots

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Theorem (a new estimate)

Let K_n denote the number of knots of n crossings. Then

$$4.45 \quad < \quad \liminf_{n\to\infty} \sqrt[n]{K_n} \quad \leq \quad \limsup_{n\to\infty} \sqrt[n]{K_n} \quad < \quad 10.4.$$

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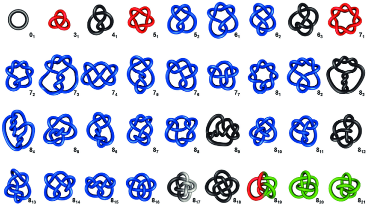
(Standard) definitions:

- A *knot* is a pair $(\mathbb{R}^3, K)$, where $K = S^1$ is a smoothly embedded circle in $\mathbb{R}^3$, considered up to homeomorphisms of pairs.
- The *crossing number* of a knot $(\mathbb{R}^3, K)$ is the smallest number of crossings (double points) of all regular plane projections of K.
- For example, $K_0 = 1$, $K_1 = 0$, $K_2 = 0$, $K_3 = 1$, $K_4 = 1$, $K_5 = 2$:

Prime knots



"Knot table of all prime knots having up to eight crossings" by S. Fielden, D. Leigh, and S. Woltering. (2017). Molecular Knots. Angewandte Chemie. 56. 10.1002/anie.201702531. (is licensed under CC BY 4.0)

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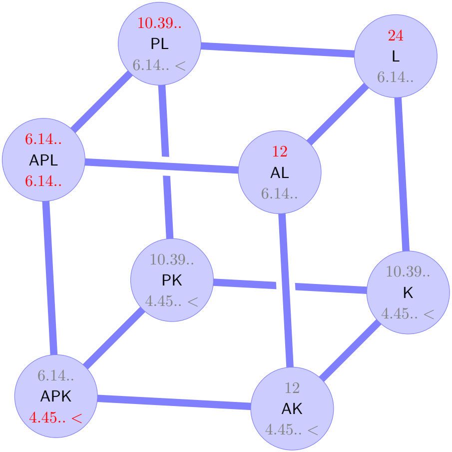
- Here, "<10.4" follows from results of Stoimenow (2004) based on Sundberg and Thistlethwaite (1998).
- "$4.45<$" is a new result based on Vershik, Nechaev, and Bikbov (2000, growth of random heaps).
- The previous known lower bound is "$2.13<$" due to Ernst and Sumners (1987).
- (Also, there is a widely known misprint "$2.68<$" by Welsh (1992).)
- The next aim is to show "$4.765<$" (in order to have a lower bound x such that $x^3 > (10.4)^2$).

Upper and lower bounds for related sequences



'K' stands for 'knots'

'L' stands for 'links'

'P' stands for 'prime'

'A' stands for 'alternating'; e. g., 'APK' means 'alternating prime knots'

Thank you