Abrarov D.L.

Mechanics and arithmetic

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The Euler-Poisson equations (EPE)

The EPE - ordinary differential equations describing the motion of three-dimensional heavy rigid bodies in a plane-parallel gravity field around a fixed point

$$\dot{\vec{M}} = \left[\vec{M}, \vec{\omega}\right] + k[\vec{\gamma}, \vec{c}]$$

$$\dot{\vec{\gamma}} = [\vec{\gamma}, \vec{\omega}]$$

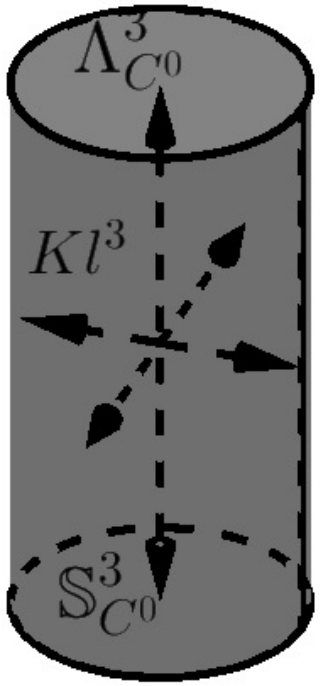
$\vec{\omega}$	–	the angular velocity of the body,
$\vec{M} = I\vec{\omega}$	–	kinetic moment of the body,
I	–	diagonalized tensor of inertia of a body at its fixed point,
$\vec{c}$	–	directing vector of a straight line from a fixed point to the center of mass,
$\vec{\gamma}$	–	the projection of the vertical vector on the axis of the moving coordinate, system rigidly connected to the body
$k = mg\lvert r_c \rvert$	–	coefficient equal to the product of body weight by the distance from a fixed point to the center of mass,
$\lvert r_c \rvert$	–	distance from a fixed point of a rotating rigid body to the center of mass.

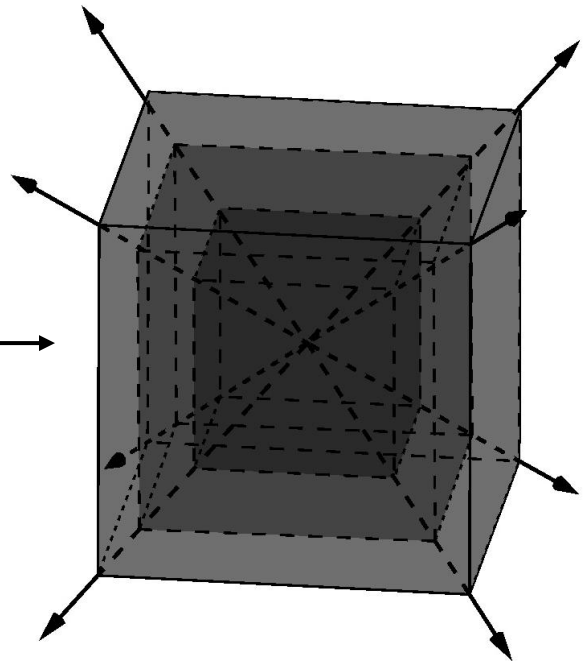
Formulation of the problem

To find:

1. **General EPE solution in a unified analytical class of functions**
2. **Unified integrability and classification mechanism for all integrable cases**
3. **Unified mechanical and geometric interpretation for all integrable cases**
4. **Experimental verification of the unified theory of EPE**
5. **Correlation of the general solution of the EPE with classical results**

Geometric models of involution $\mathbb{Z}_2^{\mathrm{E-P}}(t)$





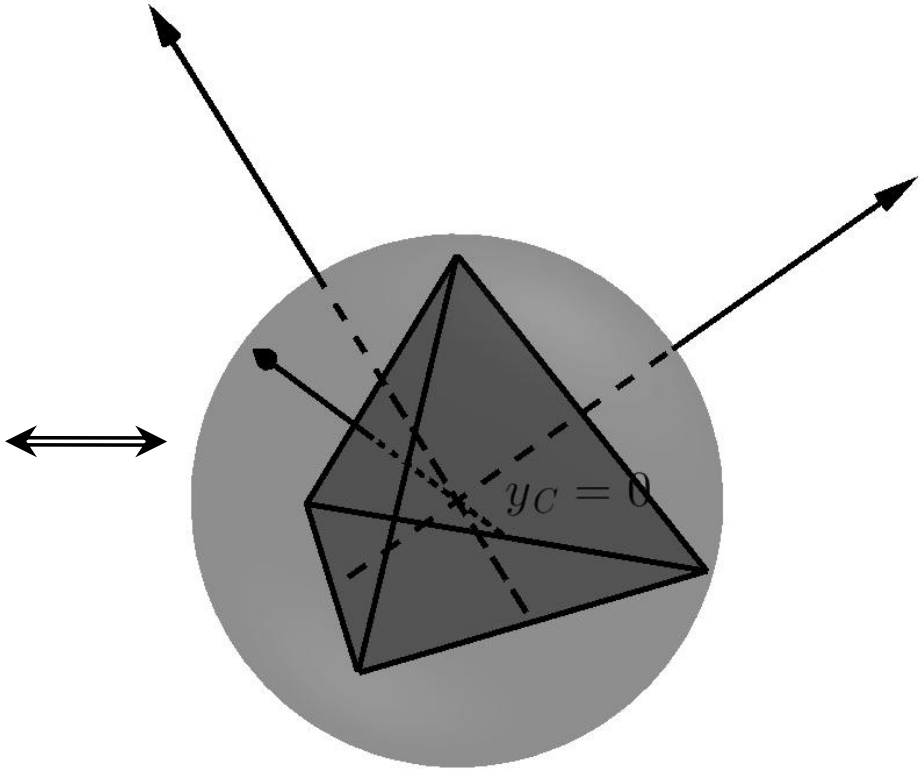


График $Kl^3_{\text{афф}}$

$T_*\mathbb{S}^3 \cong T^*\mathbb{S}^3$

$\mathbb{R} \cup \infty$

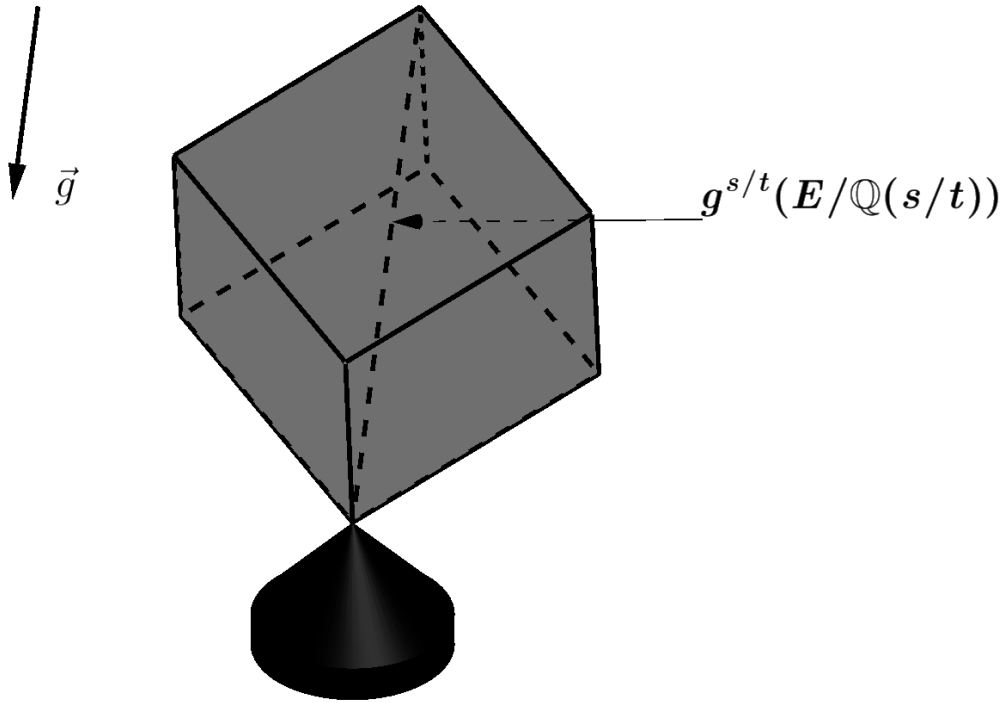
$Mom(T_c^2) \cong \begin{bmatrix} SO^{an}(3)_{eq} \\ SO^{E-P}(3) \end{bmatrix}$

$\mathbb{Z}_2^{an}(t/\mathbb{Q})$ $\Longleftrightarrow$ $\mathbb{Z}_{0,an}^{\mathbb{E}^3(\mathbb{R})/\mathbb{Z}^3}$ $\Longleftrightarrow$ $\begin{bmatrix} \mathbb{Z}_2^{an}{}_{eq}(t) \\ \mathbb{Z}_2^{E-P}(t) \end{bmatrix}$

Geometric models of involution $\mathbb{Z}_2^{\mathrm{E-P}}(t)$

- T_c^2 – tetrahedron accompanying analytical tops
- Mom – momentum map
- $Z_{0,an}^{\mathbb{E}^3/\mathbb{Z}^3} \cong (Z_0^{\mathbb{E}^3/\mathbb{Z}^3}(Sym_O, Rot_O^{\pi}))/(Sym_O \cong_{an} Rot_O^{\pi})$ – the analytic central symmetry of the lattice $\mathbb{E}^3/\mathbb{Z}^3$ centered at O
- $\mathbb{Z}_{2,an,+}^{E-P}(t) \cong Z_{0,an}^{\mathbb{E}^3/\mathbb{Z}^3} / Rot_O^{\pi}$ – additive part of involution $\mathbb{Z}_2^{E-P}(t)$
- $\mathbb{Z}_{2,an,\times}^{E-P}(t) \cong Z_{0,an}^{\mathbb{E}^3/\mathbb{Z}^3} / Sym_O$ – multiplicative part of involution $\mathbb{Z}_2^{E-P}(t)$
- $\Lambda_{C^0}^3$ – functional 3d-Lobachevsky space $\Leftrightarrow g^t{}_{S_{big,\,C^0}^{1,*}}(\mathbb{S}^3)$
- eq – equivariant

General solution of the EPE as a group law on the universal elliptic curve over $\mathbb{Q}$



Graph $d\mathbb{S}^3_{an} = dg^{s/t}_{E-P}$

$\vec{M}(t) = g^t{}_{an,eq}\, SO(3,\mathbb{R})$

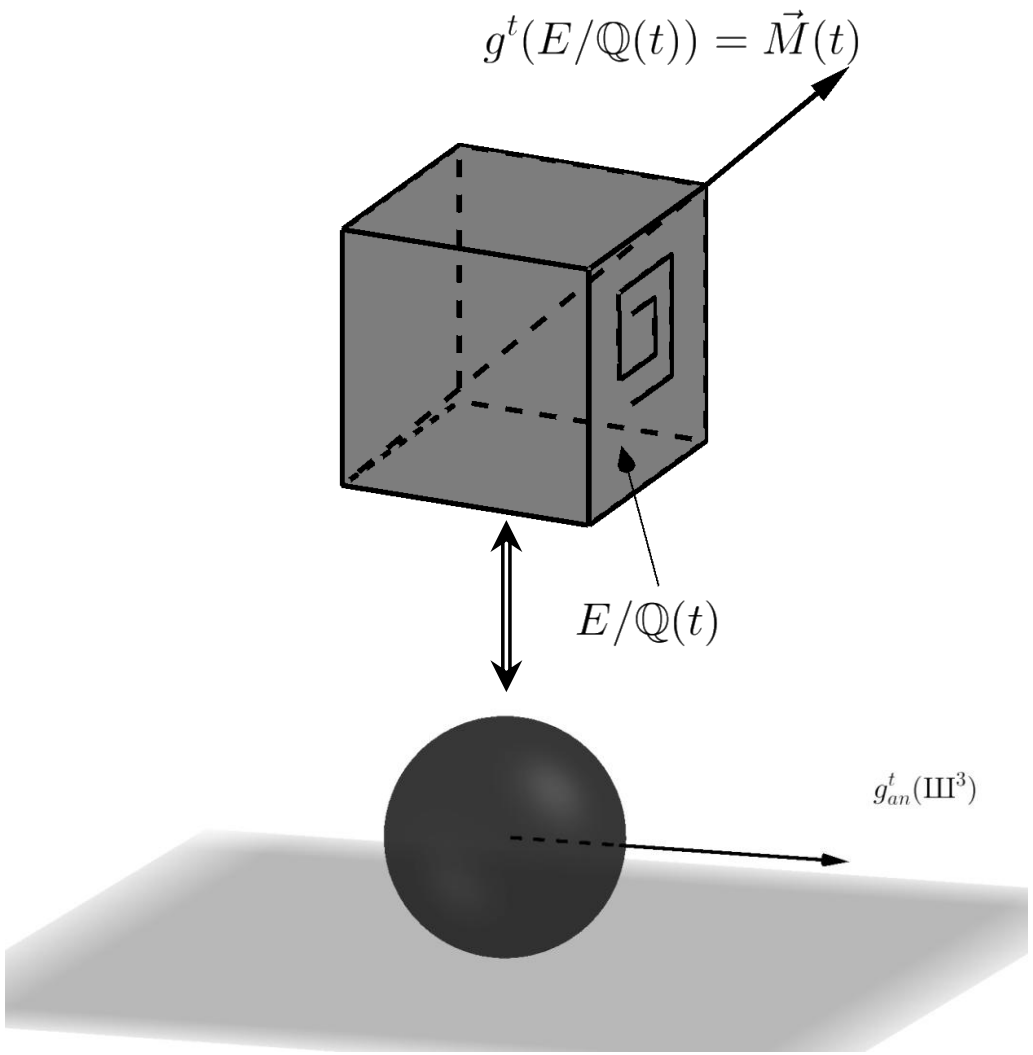
$\vec{M}(s) = g^s{}_{an,eq}\, SO(3,\mathbb{C})$

$g^{s/t}(E/\mathbb{Q}(s/t))$ – group law on the universal elliptic curve over $\mathbb{Q}$

$X_0(E/\mathbb{Q}(s/t))$
– accompanying tetrahedron $T^2_{c,*}(\mathbb{C}/\mathbb{R})$

$E/\mathbb{Q}(s/t)$
– accompanying *co*tetrahedron $T^{2,*}_c(\mathbb{C}/\mathbb{R})$

General solution of the EPE as a canonical analytic structure on SO (3)



$\vec{M}(t) = \zeta_{eq}(SO(3,\mathbb{R})) = \ Spec_{an,eq}\ SO(3,\mathbb{R}\ )$

$\vec{M}(s) = \zeta_{eq}(SO(3,\mathbb{C})) = \ Spec_{an,eq}\ SO(3,\mathbb{C})$

$\vec{M}(s) = g^s_{an}(Ш^3)$ – analytic simply connected self-duality of the 3d-ball $Ш^3(\mathbb{C})$

$\zeta_{eq}(SO(3))$ is an equivariant

- zeta-function of the variety $SO(3)$
- the global analytic structure on the group $SO(3)$

Exact solvability of the EPE

Theorem 1 (Unified theory, Abrarov D.L., 2007, 2018)

A general solution of the EPE exists and is represented by the canonical exponent of the 3d-sphere $\mathbb{S}^3(\mathbb{C})$ - a sphere with a canonical continuous metric having the potential:

$$F = \exp(t^2 - {\omega_3}^2 - {\omega_2}^2 - {\omega_1}^2 - {\gamma_3}^2 - {\gamma_2}^2 - {\gamma_1}^2)$$

and has the form of equivariant analytic structures on the groups $SO(3,\mathbb{C})$ и $SO(3,\mathbb{R})$:

$$\vec{M}(s) = \exp\big(\zeta(s, \Delta_{12}(q))\big), s \in \mathbb{C} \qquad \textbf{(1)}$$

$$\vec{M}(t) = \exp\left(\zeta\left(\frac{1}{2} + it, \Delta_{12}(q)\right)\right), t \in \mathbb{R} \qquad \textbf{(2)}$$

where the form $\Delta_{12}(q)$ is the only parabolic form of weight 12 with respect to the group $SL_2(\mathbb{Z})$:

$$\Delta_{12}(q) = q\prod_{n=1}^{\infty}(1-q^n)^{24}, \qquad q = e^{2\pi i z}, \qquad z \in \mathbb{H}^+$$

Exact solvability of the EPE

Theorem 2 (Unified integrability theory, Abrarov D.L., 2018)

The general solution of the EPE over $\mathbb{R}$-time is a group CW-complex of particular solutions represented by the functions:

$$\vec{M}(t)_{partial} = (\vec{M}(s)_{general}|_{s=t})_{CW} = \exp\left(\zeta(s, E/\mathbb{Q})\overrightarrow{\zeta_0(s, E/\mathbb{Q})}\right),$$

where

- $E/\mathbb{Q}$ belongs to the set of elliptic curves over the field of rational numbers $\mathbb{Q}$;
- $\zeta(s, E/\mathbb{Q})$ is the zeta function of the elliptic curve $E/\mathbb{Q}$;
- $\overrightarrow{\zeta_0(s, E/\mathbb{Q})} = \left((\zeta(s_1, E/\mathbb{Q}) = 0), (\zeta(s_2, E/\mathbb{Q}) = 0), (\zeta(s_3, E/\mathbb{Q}) = 0)\right)$;
- $\vec{\gamma}(t_0) = \overrightarrow{\zeta_0(s, E/\mathbb{Q})}$ (mod 3-ordered trivial zeros);
- $\vec{\omega}(t_0) = \overrightarrow{\zeta_0(s, E/\mathbb{Q})}$ (mod 3-ordered nontrivial zeros).

{Classes of $\mathbb{R}$-isogenicity of spectral curves $E/\mathbb{Q}$} $\longleftrightarrow$ {types of $\mathbb{R}$-integrable cases}

Exact solvability of the EPE

Theorem 3 (Equivariant KAM-theory, Abrarov D.L., 2007, 2019)

The general solution of the EPE over $\mathbb{C}$-time is a group functional CW-complex of particular solutions represented by the functions:

$$\vec{M}(s)_{partial} = (\vec{M}(s)_{general})_{CW} = \exp\left((L(s, E/\mathbb{Q})\overrightarrow{L_0(s, E/\mathbb{Q})}\right),$$

where

- $E/\mathbb{Q}$ belongs to the set of elliptic curves over the field of rational numbers $\mathbb{Q}$;
- $L(s, E/\mathbb{Q})$ – L-function of the elliptic curve $E/\mathbb{Q}$;
- $\overrightarrow{L_0(s, E/\mathbb{Q})} = \left((L(s_1, E/\mathbb{Q}) = 0), (L(s_2, E/\mathbb{Q}) = 0), (L(s_3, E/\mathbb{Q}) = 0)\right)$;
- $\vec{\gamma}(t_0) = \overrightarrow{L_0(s, E/\mathbb{Q})}$ (mod 3-ordered trivial zeros);
- $\vec{\omega}(t_0) = \overrightarrow{L_0(s, E/\mathbb{Q})}$ (mod 3-ordered nontrivial zeros).

{Classes of $\mathbb{C}$-isogenicity of spectral curves $E/\mathbb{Q}$} $\longleftrightarrow$ {types of $\mathbb{C}$-integrable cases}

The general solution of the EPE as the canonical self-duality of the 3d-sphere $\mathbb{S}^3$

$$F \coloneqq \{\mathrm{T}_* \, \mathbb{S}^3[\vec{\gamma}] \ \cong T^* \, \mathbb{S}^3[\vec{\omega}]\} = \ g^{t}_{S^1_{big,c^0}}(\,\mathbb{S}^3)$$

$$F_{\mathbb{R},\text{общ}} = F_{kow} = exp_{S^1_{big,\,c^0}}(\mathbb{S}^3(\mathbb{R})) \Rightarrow \vec{M}_{\text{общ}}(t) = exp_{S^1_{big,\,c^0}}(\mathbb{S}^2(\mathbb{R})) = (\mathbf{2})$$

$$F_{\mathbb{C},\text{общ}} = F_{kow} \ \otimes \mathbb{C} = exp_{S^1_{big,\,c^0}}(\mathbb{S}^3(\mathbb{C})) \Rightarrow \vec{M}_{\text{общ}}(s) = exp_{S^1_{big,\,c^0}}(\mathbb{S}^2(\mathbb{C})) = (\mathbf{1})$$

The general solution of the EPE as the canonical self-duality of the 3d-sphere $\mathbb{S}^3$

$$\boldsymbol{F_{\mathbb{R},general} = F_{kow} = \left|\,(p+iq)^2+(\gamma_1+i\gamma_2)\right|^2} \;\&\; \boldsymbol{F_{\mathbb{C},general} = \left|\,(p+iq+jr)^2+(\gamma_1+i\gamma_2+j\gamma_3)\right|^2}$$

– ***the potentials of the phase*** ($\vec{\gamma}$-$\vec{\omega}$)***-solidity:*** the potentials of the analytic dynamics of the tetrahedra

accompanying the tops – $T_c^{2,*}(\mathbb{R})$ & $T_c^{2,*}(\mathbb{C})$ correspondingly,

where $\{\, i^2 = j^2 = -1,\ i+j = i\cdot j \,\}$ – the equations of the accompanying *co*trihedron $Tr_c^2(\mathbb{C})$

- $F_{\mathbb{R},general} = \{const\} \Leftrightarrow \Lambda_{an}^5(\mathbb{R})[\vec{\omega}_{an}(t), \vec{\gamma}_{an}(t)]$ – orbit of a *co*tetrahedron $T_c^{2,*}(\mathbb{R})$

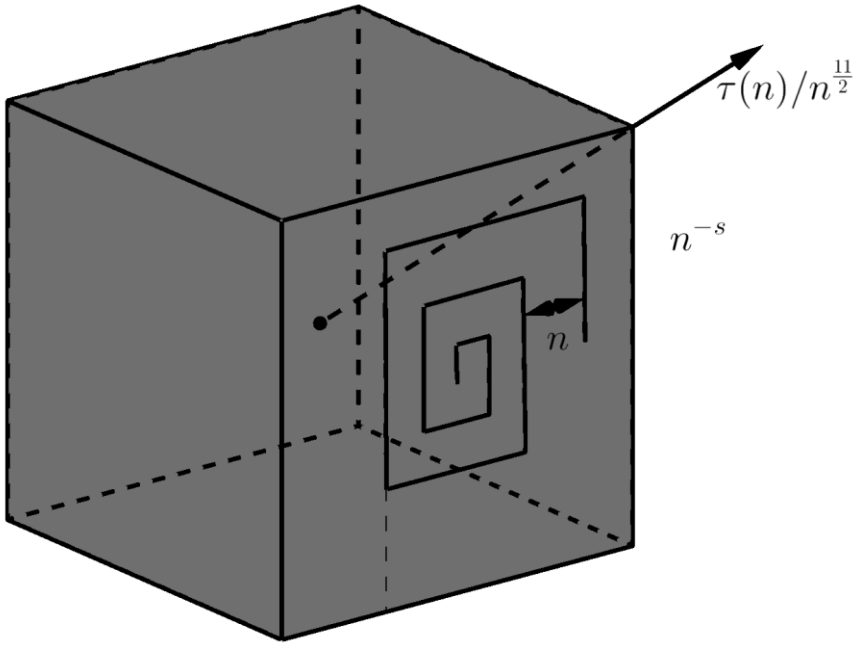
- $F_{\mathbb{C},general} = \{const\} \Leftrightarrow \Lambda_{an}^5(\mathbb{C})[\vec{\omega}_{an}(s), \vec{\gamma}_{an}(s)]$ – orbit of a *co*tetrahedron $T_c^{2,*}(\mathbb{C})$,

где Λ_{an}^5 – Analytical functional 5d-Lobachevsky space

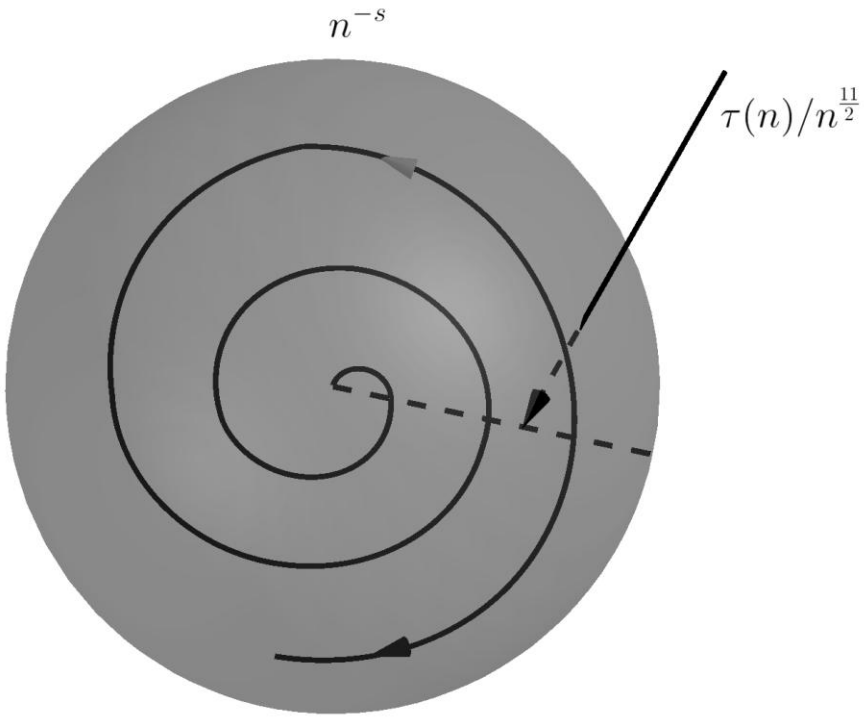
– Orbit of the flow of large (co)circles on the sphere $\mathbb{S}^5$

– *Orbit of canonical ordered* $\mathbb{S}^3$*-self-duality* $\{T^*\,\mathbb{S}^3[\vec{\omega}] \cong T_*\,\mathbb{S}^3[\vec{\gamma}]\}$

Geometric model $\zeta_n(\mathbf{s}, \Delta_{12}(q))$, $\zeta(\mathbf{s}, \Delta_{12}(q))$ and canonical norm of the EPE phase flow



$\Longleftrightarrow$



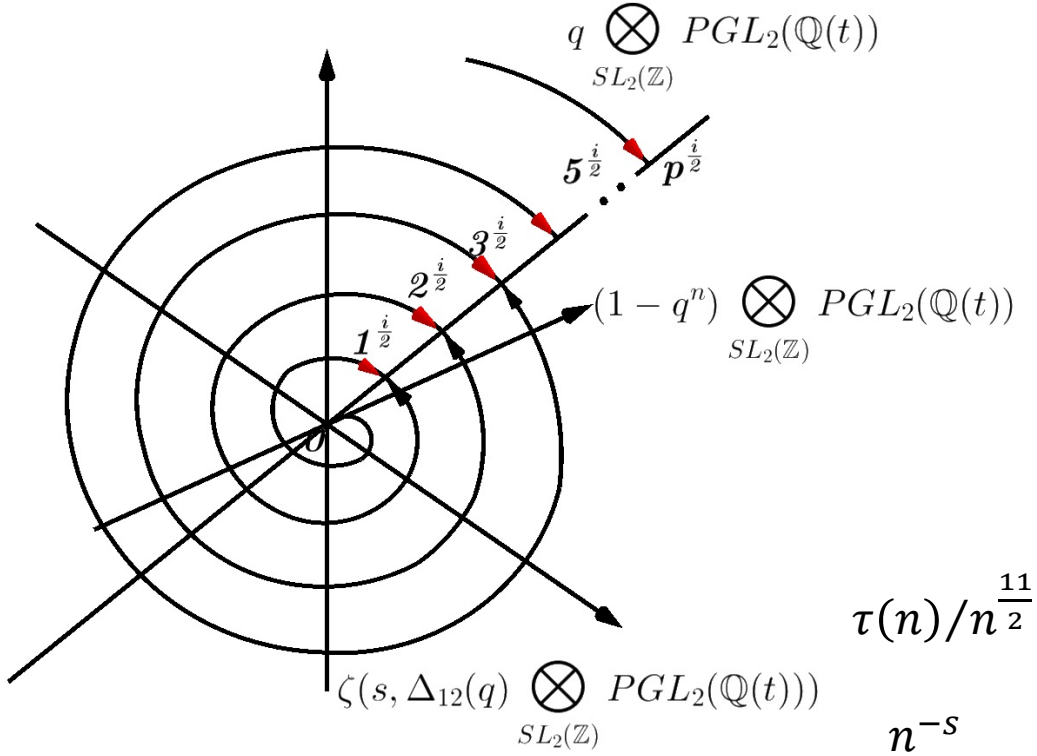
$SO\left(3, A_{\mathbb{Q}(s)}\right)$ - model $\mathbb{S}^3_{C^0}$

$PGL_2\left(A_{\mathbb{Q}(s)}\right)$ - model $\mathbb{S}^3_{C^0}$

Definition: $A_{\mathbb{Q}(s)} \coloneqq \mathbb{Q}(s) \otimes^{\mathbb{P}}_{p^{\frac{i}{2}}} \mathbb{Q}_p(s)$ [$= it(mod\ id\ \mathbb{Z}^{\mathrm{E-P}}_{2,CW}(s))$] is the canonical norm on $S^1_{big,\,C^0}$ $(\mathbb{S}^3)$

Additive representation $\zeta(s, \Delta_{12}(q))$: $\zeta(s, \Delta_{12}(q)) \coloneqq \sum_{n=1}^{+\infty} \frac{\tau(n)/n^{\frac{11}{2}}}{n^s}$

Zeta function $\zeta(s, \Delta_{12}(q))$ as a global continuous metric on the 3d-sphere $\mathbb{S}^3$



Notation: $CW(n)$ - subcomplex of level n in CW-complex $\left(\mathbb{S}^3_{C^0}\right)_{CW}$

$$\left(\mathbb{S}^3_{C^0}\left(\zeta(s, \Delta_{12}(q))\right)\right)_{CW}$$

$$\Updownarrow$$

$$\tau(n)/n^{\frac{11}{2}} = \left(Trace\ PGL_2\left(A_{\mathbb{Q}(s)}\right)\right)_{CW(n)} = \left(\mathbb{S}^3_{C^0}\right)_{CW(rk=0)} = \left(\mathbb{S}^0_{big}\right)_{CW(n)}$$

$$n^{-s} = \left(Det\ PGL_2\left(A_{\mathbb{Q}(s)}\right)\right)_{CW(n)} = \left(\mathbb{S}^3_{C^0}\right)_{CW(rk=1)} = \left(\mathbb{S}^1_{big}\right)_{CW(n)}$$

$$\left(\tau(n)/n^{\frac{11}{2}}\right) \cdot n^{-s} = \left(Discr\ PGL_2\left(A_{\mathbb{Q}(s)}\right)\right)_{CW(n)} = \left(\mathbb{S}^3_{C^0}\right)_{CW(rk=2)} = \left(\mathbb{S}^2_{big}\right)_{CW(n)}$$

$$«+\left(\tau(n)/n^{\frac{11}{2}}\right) \cdot n^{-s} + » = \left(Discr\ PGL_2\left(A_{\mathbb{Q}(s)}\right)\right)_{CW(n)_{group}} = \left(\mathbb{S}^3_{C^0}\right)_{CW(rk3)} = \left(\mathbb{S}^3_{C^0}\right)_{CW(n)}$$

Geometry, dynamics and arithmetic of the global invariant $\boldsymbol{F}$

$\boldsymbol{F}$ is the canonical global (over $\mathbb{C}\cup\infty$) C^0-metric on the sphere $\mathbb{S}^3(\mathbb{C})$

is the potential of the canonical C^0-duality $\{\mathrm{T}_*\ \mathbb{S}^3[\vec{\gamma}]\ \cong T^*\ \mathbb{S}^3[\vec{\omega}]\}$

$\boldsymbol{F}$ is the potential:

- of the canonical C^0-duality $T^2_{c,*}(\mathbb{R}) \leftrightarrow T^{2,*}_c(\mathbb{R})$ of the accompanying *bi*tetrahedron
- of the equivariant C^0-continuation of classical solutions (including the "Euler separatrix") to $s = \infty$
- *of the bimodular parametrization* $\{X_0(N) \leftrightarrow E/\mathbb{Q}\}$ *curves over* $\mathbb{Q}\Leftrightarrow$ *property of phase* ($\vec{\gamma}$ - $\vec{\omega}$) - *solidity*
- of the domain of the canonical 3d-exponent is globally (over $\mathbb{C} \cup \infty$) an analytic function e^z

Geometry, dynamics and arithmetic of the global invariant *exp F*

exp F – is the canonical global ("over" $\mathbb{C}\cup\infty$) C^1-metric on the sphere $\mathbb{S}^3(\mathbb{C})$

– is the potential of the canonical global C^1-duality $\{\mathrm{T}_* \, \mathbb{S}^3[\vec{\gamma}] \cong T^* \, \mathbb{S}^3[\vec{\omega}]\}$

***exp F* is the potential:**

- of the equivariant C^{an} (global C^1) - continuation of classical solutions to $s = \infty$
- Φ is the potential of the derived bimodular parametrization $\{X_0(N) \leftrightarrow E/\mathbb{Q}\} \Leftrightarrow$ property of the derived phase ($\vec{\gamma}$ - $\vec{\omega}$)-solidity
- Φ is the potential of the canonical equivariant (global class C^1) 3d-exponent $e^{z\cup_{eq}\infty}$
- Φ - potential of finitely generated determinization of classical chaos

Mechanical meaning of the global invariant $\boldsymbol{F}$

$$\boldsymbol{F} = \exp\left(\boldsymbol{t}^2 - \boldsymbol{\omega_3}^2 - \boldsymbol{\omega_2}^2 - \boldsymbol{\omega_1}^2 - \boldsymbol{\gamma_3}^2 - \boldsymbol{\gamma_2}^2 - \boldsymbol{\gamma_1}^2\right)$$

$\boldsymbol{F}$ *is the canonical metric of the phase space* $P \cong \mathbb{R}^{6+1}(\vec{\gamma}(t), \vec{\omega}\,(t))/\,\mathbb{Z}_{2,C^0}^{\mathrm{E-P}}(t)$ *of the EPE equations*

- $\boldsymbol{F}$ is the Hamiltonian of the trivial top ($I = E, \vec{c} \neq 0$)

- $\boldsymbol{F}$ is the vertical equilibrium Hamiltonian of the analytical 3d-pendulum (AMP)

$rk\ P =$ number of degrees of freedom AMP = $rk(Z_{0,C^0}^{\mathbb{E}^3/\mathbb{Z}^3})$

- $Z_{0,C^0}^{\mathbb{E}^3/\mathbb{Z}^3}$ *is the central* C^0*−symmetry in* $\mathbb{E}^3(\mathbb{C})$ *with the center* O

$$rk(Z_{0,C^0}^{\mathbb{E}^3/\mathbb{Z}^3}/ZentrZ_{0,C^0}^{\mathbb{E}^3/\mathbb{Z}^3}) = ord(Z_{0,C^0}^{\mathbb{E}^3/\mathbb{Z}^3}/ZentrZ_{0,C^0}^{\mathbb{E}^3/\mathbb{Z}^3})_{CW}[pt] =$$

$$= rk\ \mathbb{Z}_{2,C^0}^{\mathrm{E-P}}(t) = (0^0 + 1^1 + 2^2 + 3^3 + 4^4) - 2^3 = 281$$

Mechanical meaning of the global invariant $exp\ F$

$$\boldsymbol{exp\ F} = \left| (\boldsymbol{p} + \boldsymbol{iq} + \boldsymbol{jr})^2 + (\boldsymbol{\gamma_1} + \boldsymbol{i\gamma_2} + \boldsymbol{j\gamma_3}) \right|^2 \ \&\ \{\ \boldsymbol{i}^2 = \boldsymbol{j}^2 = -\mathbf{1},\ \boldsymbol{i} + \boldsymbol{j} = \boldsymbol{i} \cdot \boldsymbol{j}\ \}$$

$\boldsymbol{exp\ F}$ – *is the canonical metric of the phase flow* g^S_{E-P} *of the EPE*

- $\boldsymbol{exp\ F}$ is the Hamiltonian of the general top ($I \neq E, \vec{c} \neq 0$)
- $\boldsymbol{exp\ F}$ is the Hamiltonian of self-oscillations of the analytical 3d-pendulum (AMP) around its vertical equilibrium

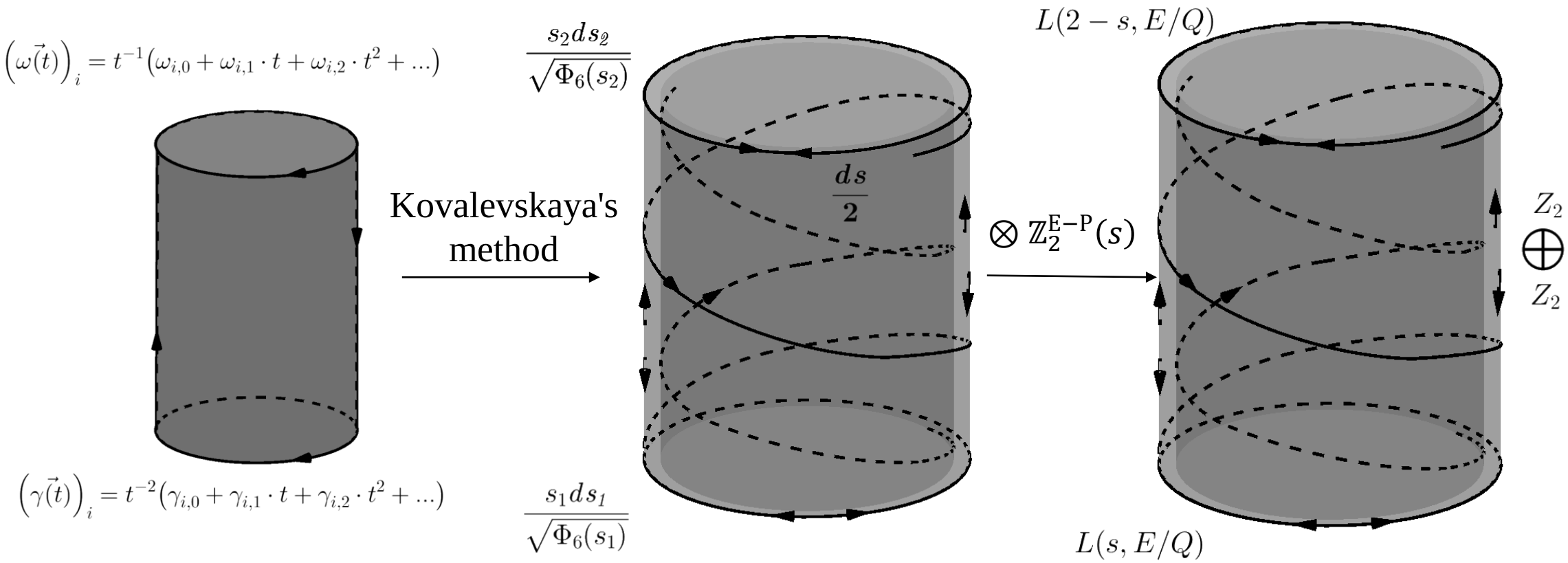
$rk\ g^S_{E-P}/P =$ *number of phase states of the EPE equations* $= ord(Z^{\mathbb{E}^3/\mathbb{Z}^3}_{0,an})[pt] =$
$rk\ \mathbb{Z}^{\mathrm{E-P}}_{2,C^{an}}(t) = e^{281}$

- pt is an arbitrary point in the Euclidean space

Numbers 281, e^{281} *were discovered by Ya.V. Ryazantsev in the context of the quantum loop gravity theory*

Comment. The 3d-pendulum with Hamiltonian $\boldsymbol{exp\ F}$ is the canonical analytic pendulum.

Application of the theory: model of the generalized Dzhanibekov's effect



Dzhanibekov effect: $E/\mathbb{Q} = E^{min}/\mathbb{Q} = \{y^2 = x^3 - x\}$

For computer visualization of the Dzhanibekov 's effect see [5]

Application of the theory: a model of the generalized Dzhanibekov's effect

$\Phi_6(s)$ is the spectral polynomial of the Goryachev-Chaplygin case

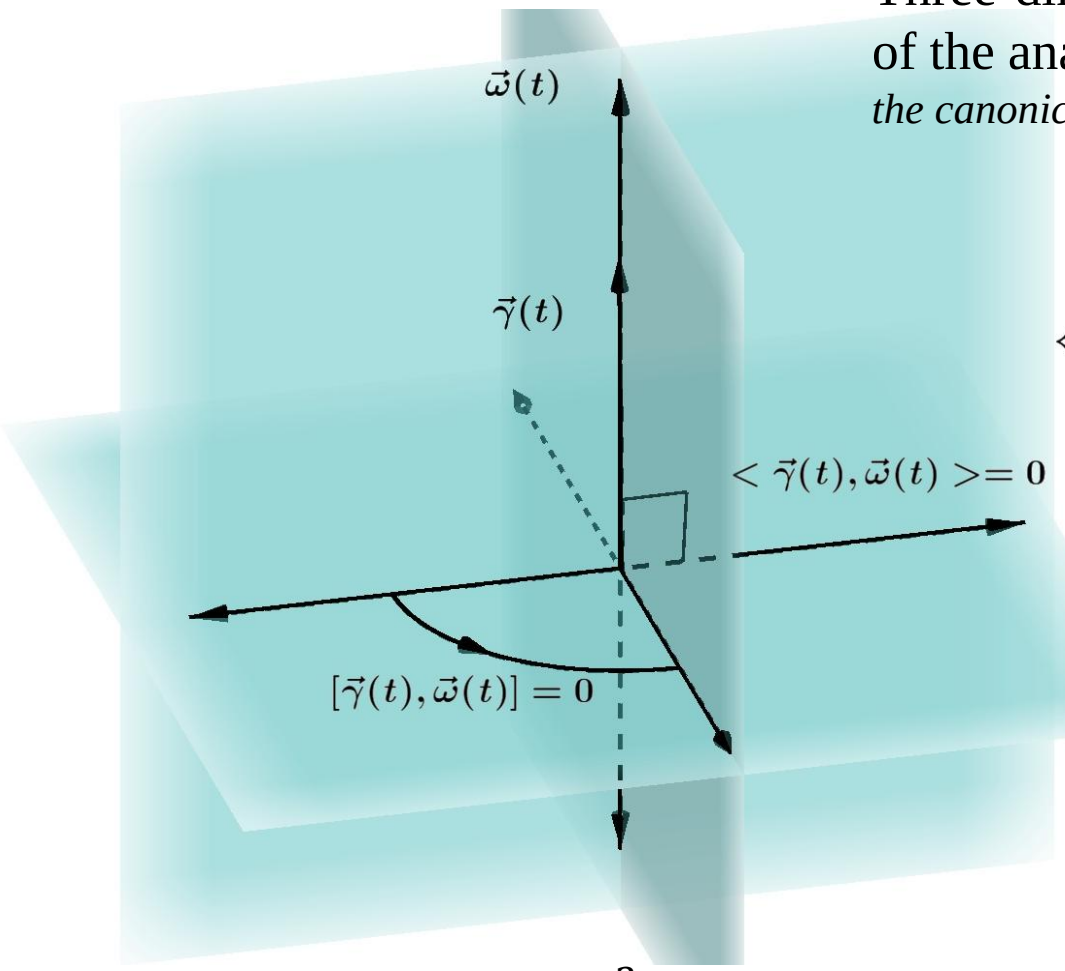
$\mathbb{Z}_2 \oplus \mathbb{Z}_2 \cong Ш\mathrm{a}\left(E^{min}/\mathbb{Q}\right)$ (V. A. Kolyvagin)

Conjecture:

$$\{\text{generalized Dzhanibekov's effect}\} \Leftrightarrow \{\text{classes of solutions of the EPE}\}$$
$$\updownarrow$$
$$\{\ Ш\mathrm{a}(E/\mathbb{Q})\ \}$$

$Ш\mathrm{a}(E/\mathbb{Q})$ – Shafarevich – Tate groups of curves $\mathrm{E}/\mathbb{Q}$

The model of the 3d-Klein bottle as a configuration manifold of the analytical 3d-pendulum in vertical equilibrium



Affine Graph $\mathrm{Kl}^3 \Leftrightarrow [\,,]_{eq}$

Three-dimensional Klein bottle Kl^3 is the configuration space of the analytical 3d-pendulum (AMP) in vertical equilibrium (*equivalent to the canonical analytic continuation of the classical pendulum phase flow at* $t = \infty$)

The functional manifold Kl^3 can be given by the system of equations:

$$\Leftrightarrow \begin{cases} < \vec{\gamma}(t), \vec{\omega}(t) >= 0 \\ [\vec{\gamma}(t), \vec{\omega}(t)] = 0 \end{cases}$$

Minimal mechanical (pendulum) model g^{s}_{E-P}:

- *phase flow AMP*
- *<u>self-oscillations of AMP around its vertical equilibrium</u>*
- *the space of AMP exits from its vertical equilibrium*

The fundamental mistake of the classics

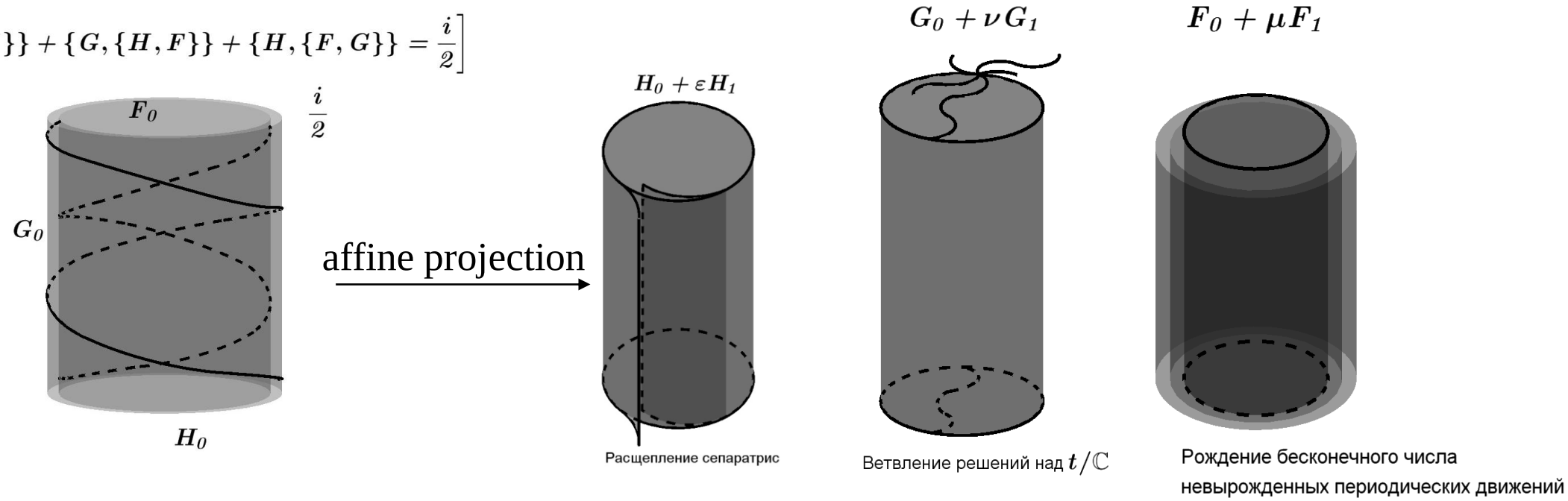
The classical solutions (defined over $\mathbb{R}$) are $\mathbb{Z}_{2,+}^{\boldsymbol{E-P}}(\boldsymbol{t})$-non-invariant, where $\mathbb{Z}_{2,+}^{\boldsymbol{E-P}}(\boldsymbol{t}) \cong \mathbb{Z}_{2}^{\mathrm{E-P}}(\boldsymbol{t})|_{\mathbb{R}_+} \subset \mathbb{Z}_{2}^{\mathrm{E-P}}(\boldsymbol{t})$. This excludes the points $\boldsymbol{t} = \boldsymbol{0}, \infty$ from the of flow $\boldsymbol{g}_{\boldsymbol{E-P}}^{\boldsymbol{t}}$ domain definition and leads to the loss of **the phase ($\vec{\boldsymbol{\gamma}}$-$\vec{\boldsymbol{\omega}}$)-solidity property**, inducing also the following mistakes:

- *Non-equivariance of gluing the signs of classical solutions quadratures*
- *Non-equivariance of classical solutions* (***but not integrals and invariant relations***)
- *Non-equivariance of classical geometric interpretations of integrability cases*
- Destruction of the tetrahedron and trihedron accompanying the tops, i.e. the tops themselves
- *Omitting angular velocity of the <u>analytic</u> tops*
- Omitting the canonical global group geometric model of the flow g_{E-P}^{t}
- *Omitting the additional (4th) integral F*
- Omitting the general integral $F_{\mathbb{C},general}$
- Omitting the constructive general solution
- Omitting nature of the Kovalevskaya's case and method
- *Non-equivariance of the KAM theory for the EPE*
- *Non-equivariance of non-integrability results for the EPE*
- *Non-equivariance of Dzhanibekov's effect theoretical explanation*

The fundamental mistake: the classics omits globally analytical time t/$\mathbb{Z}_{2,an}^{\boldsymbol{E-P}}(\boldsymbol{t})$ - hypothetically - real time

The chaos-determinism ratio for the EPE

$$\left[\{F,\{G,H\}\}+\{G,\{H,F\}\}+\{H,\{F,G\}\}=\frac{i}{2}\right]$$



Global Atlas on g^s_{E-P} $\xleftarrow{\otimes\, \mathbb{Z}_2^{\mathrm{E-P}}(s)}$ Affine atlas on g^s_{E-P}

[Finitely generated high-dimensional determinism] [Classic chaos]

The chaos-determinism ratio for the EPE

- g^{s}_{E-P} is the phase flow of the EPE
- $\{F, G, H\}$ are a complete sets of integrals of integrable cases of the EPE
- $\{F,\{G,H\}\} + \{G,\{H,F\}\} + \{H,\{F,G\}\} = \frac{i}{2}$ – equivariant Jacobi identity
- F_0, G_0, H_0 is the complete set of integrals of the Euler case

Statement (Abrarov D.L., 2020) The global dynamics g^{s}_{E-P} is represented by a finitely generated module – by the orbit of the adjoint representation of group $Z^{\mathbb{E}^3/\mathbb{Z}^3}_{0,C^0}$ of rank 281. The symmetry $Z^{\mathbb{E}^3/\mathbb{Z}^3}_{0,C^0}$ is the canonical equivariant continuous extension of the exeptional Lie group $E_8(\mathbb{C})$.

The ratio between the general solution of the EPE and classical solutions

$$\zeta\big(s,\Delta_{12}(q)\big)=\frac{1}{2}\int_{PGL_2\left(A_{\mathbb{Q}(s/t)}\right)}\left(t^{\frac{s}{2}-1}+t^{\frac{1-s}{2}-1}\right)\left(\Theta(0,it)-\frac{1}{2}\right)dt-\frac{1}{s}-\frac{1}{1-s},$$

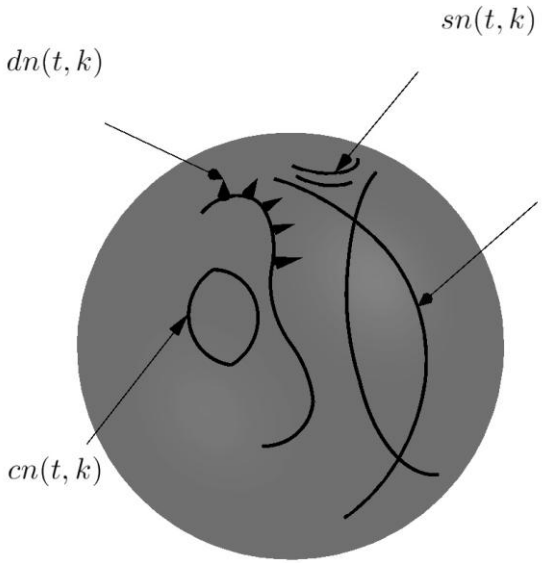
где $\Theta(z)=\frac{1}{2}+\sum_{n=1}^{+\infty}e^{-\pi i n^2 z}, z\in\mathbb{H}^+$

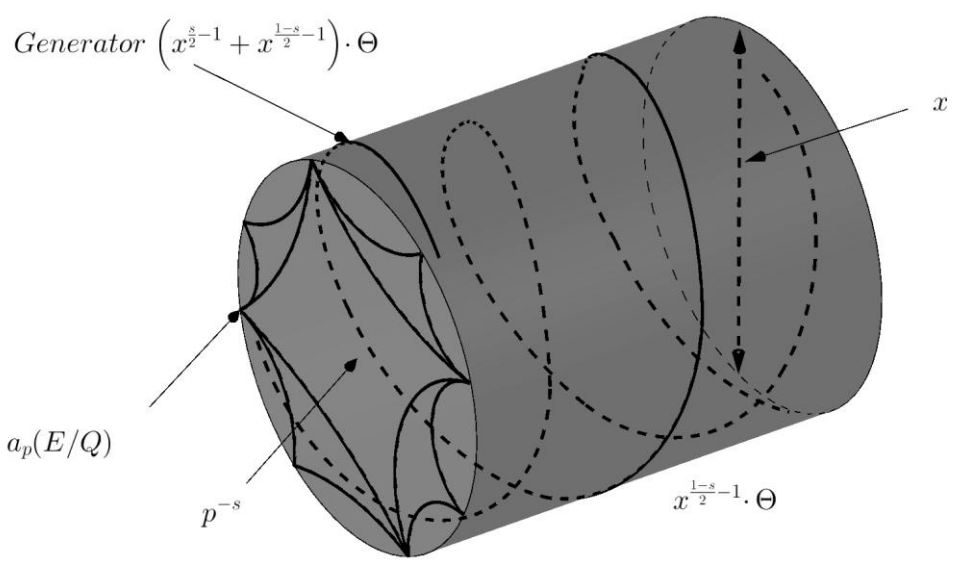
$\Theta(q)$ has the meaning:

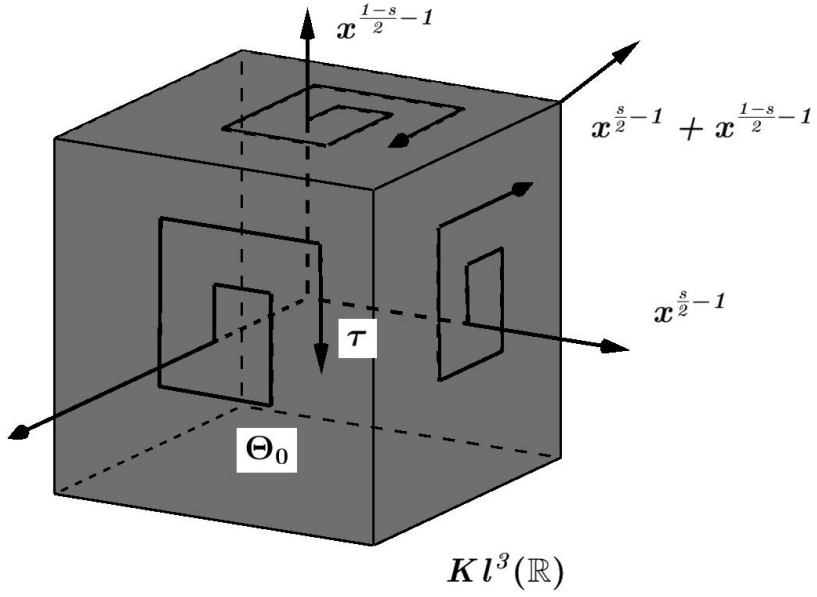
- of the canonical coordinate on the separatrix of the Euler case
- of the canonical coordinate in the connected map of the canonical affine atlas on the 3d-sphere $\mathbb{S}^3$
- of the canonical affine coordinate on the curve $E^{univ}/\mathbb{Q}$

$PGL_2\left(A_{\mathbb{Q}(s/t)}\right)$ has the meaning C^0 - invariant continuation of the separatrix of the Euler case to $s=\infty$ by the involution $\mathbb{Z}_2^{E-P}(s)$

The ratio between the general EPE solution and classical solutions







Separatrix meaning of the function $\Theta(0, it)$ in the Euler case

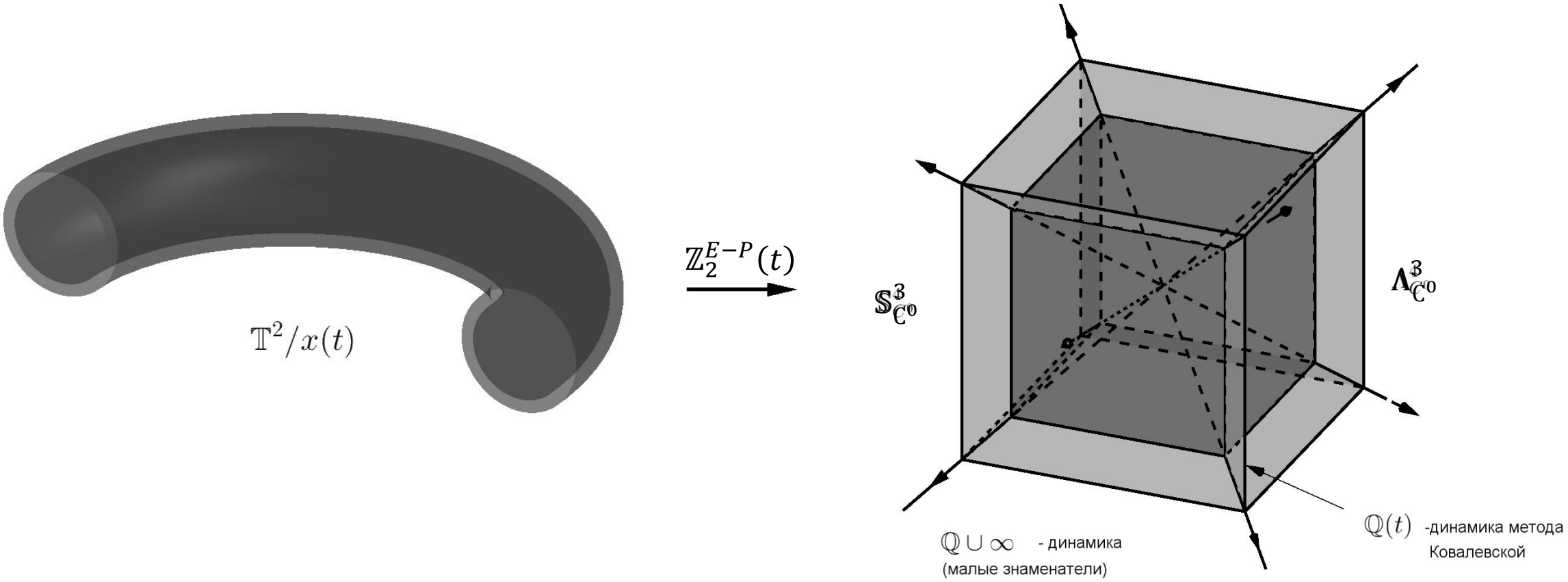
Equivariant cylindrical model of the connection "theta-zeta"

Geometric model of the connection "theta-zeta" as coordinates on the 3d-Klein bottle

Classical solutions are affinely (over $\mathbb{R}$) - equivariant, but $\mathbb{Z}_2^{E-P}(t)$ - non-equivariant

Classical integrals $\mathbb{Z}_2^{E-P}(t)$- equivariant

Geometric model of the ratio KAM - AntiKAM



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