How many roots of a system of random Laurent polynomials are real?

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In 1943 Mark Kac proved that the expected number of zeros of a random real polynomial of degree N asymptotically equals $\frac{2}{\pi}\log N$; see [Ka]. Laurent polynomial in $(\mathbb{C}\setminus 0)^n$ is called a real Laurent polynomial if its values on a compact subtorus of complex torus $(\mathbb{C}\setminus 0)^n$ are real. A zero of such polynomial on a compact subtorus is called "real zero". It turns out that the average fraction of real zeros of a random real Laurent polynomial of increasing degree N converges to not 0 but to $1/\sqrt{3}$ (> 0.5 !); see for example [ADG]. We prove that the phenomenon of nonzero fraction of real roots remains valid in many variables.

References

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