Parallel algorithm for calculating dynamic characteristics automatic control system

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Abstract. We discuss a parallel algorithm for calculating dynamic characteristics automatic control system. The developed algorithm will be used as a library of the computer algebra system (CAS) "Math Partner" [1,3,5].

Introduction

Currently, the number of automated objects in various technical areas is constantly expanding and more and more complex control objects are being automated. Analysis and calculation of the characteristics of automatic control systems (ACS) is an important scientific - practical problem of high relevance and great importance in scientific research and engineering calculations. It is planned to develop a parallel algorithm for calculating the dynamic characteristics of automatic control systems for complex objects. This algorithm will use ready-made algorithms that are described in scientific articles. [2,4,6,7,8,9,10,11]

Statement of the problem

Multichannel automatic control systems are usually called automatic control systems, which have more than one input control system. Objects that have more than one controllable value are called multichannel objects or multichannel control objects. Multichannel systems and objects are called linear and stationary. Let $y_1, ..., y_p$ be output variables, $u_1, ..., u_m$ are control parameters, and $f_1, ..., f_l$ are disturbing ones. The equations of multichannel stationary linear systems and objects can be written as the following system:

$$\sum_{j=1}^{p} a_{ij}(p)y_i = \sum_{j=1}^{m} b_{ij}(p)u_j + \sum_{j=1}^{l} c_{ij}(p)f_j, i = 1, ..., p. \qquad (1)$$

Here $a_{ij}(p), b_{ij}(p), c_{ij}(p)$ denote polynomials of the differentiation operator with constant coefficients. Passing in both parts of (1). to Laplace images with zero initial conditions, we obtain a system of algebraic equations:

$$\sum_{j=1}^{p} a_{ij}(s)Y_i(s) = \sum_{j=1}^{m} b_{ij}(s)U_j(s) + \sum_{j=1}^{l} c_{ij}(s)F_j(s), i = 1, ..., p, \tag{2}$$

where $Y_j(s) = L\{y_j(t)\}, U_j(s) = L\{u_j(t)\}, F_j(s) = L\{f_j(t)\}$. To describe multichannel systems and objects, as in single-channel systems, transfer functions are used. The transfer function $W^u_{ij}(s)$ (in Laplace images) for the j-th control parameter and the i-th output is the ratio [11]:

$$W^u_{ij}(s) = \frac{Y_i(s)}{U_j(s)}. \tag{3}$$

This transfer function can be calculated in the following way: we equate to zero in system (2) the images of all disturbing influences and control parameters, except for $U_j(s)$. From the resulting system of algebraic equations, we find the solution $Y_i(s)$, then, dividing it by $U_j(s)$, we obtain the transfer function. Similarly, the transfer function $W^f_{ij}(s)$ is determined for the j-th disturbing action and the i-th output:

$$W^f_{ij}(s) = \frac{Y_i(s)}{F_j(s)}. \tag{4}$$

In the case of multichannel systems, a complete description requires $p \cdot m$ control transfer functions and $p \cdot l$ disturbance transfer functions. These functions are written in matrix form:

$$W^u(s) = \begin{bmatrix} W^u_{11}(s) & ... & W^u_{1m}(s) \\ ... & ... & ... \\ W^u_{p1}(s) & ... & W^u_{pm}(s) \end{bmatrix}; \tag{5}$$

$$W^f(s) = \begin{bmatrix} W^f_{11}(s) & ... & W^f_{1l}(s) \\ ... & ... & ... \\ W^f_{p1}(s) & ... & W^f_{pl}(s) \end{bmatrix}; \tag{6}$$

Matrix (5) is called the control transfer function matrix or control transfer matrix. Matrix (6) is called the perturbation transfer function matrix or disturbance transfer matrix.

Let one control parameter be a delta function: $u_j = \delta(t)$, and the rest of the control parameters and disturbance are equal to zero. In this case, the solution of system (2) with zero initial conditions is denoted by $w^u_{1j}(t), w^u_{2j}(t), ..., w^u_{pj}(t)$. These functions are called weighting or impulse transient functions. The function $w^u_{ij}(t)$ describes the response of the system at the i-output when acting at the point of application of the j-th control parameter of a single impulse and is called an impulse transient or weight function for the j-th control parameter and the i-th output.

The matrix $W^u(t) = \begin{bmatrix} W^u_{11}(t) & ... & W^u_{1m}(t) \\ ... & ... & ... \\ W^u_{p1}(t) & ... & W^u_{pm}(t) \end{bmatrix}$, composed of control weighting functions is called an impulse transitional or control weighting matrix.

The impulse transition or weight matrix for the perturbation is determined in a similar way: $W^f(t) = \begin{bmatrix} W^f_{11}(t) & ... & W^f_{1l}(t) \\ ... & ... & ... \\ W^f_{p1}(t) & ... & W^f_{pl}(t) \end{bmatrix}$. Here $w^f_{1j}(t), w^f_{2j}(t), ..., w^f_{pj}(t)$ is the solution of system (1), when $f_j = \delta(t)$, and all other disturbing influences and parameters are equal to zero.

Parallel algorithm

The parallel algorithm consists of 3 steps. The cluster has n processors. We assume that the processor with number 0 is a root.

Step 1. Direct Laplace transform of a system of differential equations describing a multichannel automatic control system. As a result of the Laplace transform, we obtain the algebraic system (2) linear equations with polynomial coefficients. We use algorithms [8, 11].

Step 2. Solving the algebraic system and calculating the transfer function. We use parallel algorithms [8, 11].

Step 3. Inverse Laplace transform calculation of the transition function. We use parallel algorithms [8, 11].

Conclusion

Currently, the CAS 'Math Partner" system has built-in calculated dynamic characteristics for single-channel ACS. It is assumed that on the basis of this and using algorithms for solving linear equations with constant system coefficients in the CAS "Math Partner", algorithms will be developed for the following dynamic characteristics of multichannel ACS:
1) transfer matrix for control;
2) perturbation transfer matrix;
3) weight matrix for control;
4) perturbation weight matrix.

In addition, the CAS "Math Partner" has an interface for solving problems on the ISP RAS cluster. Using this interface allows you to return the results of calculations to the user of the CAS "Math Partner" in a familiar graphical form. We plan to supplement the package of parallel programs and make it possible to calculate the dynamic characteristics of a large-scale ACS on a cluster with distributed memory.

References

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