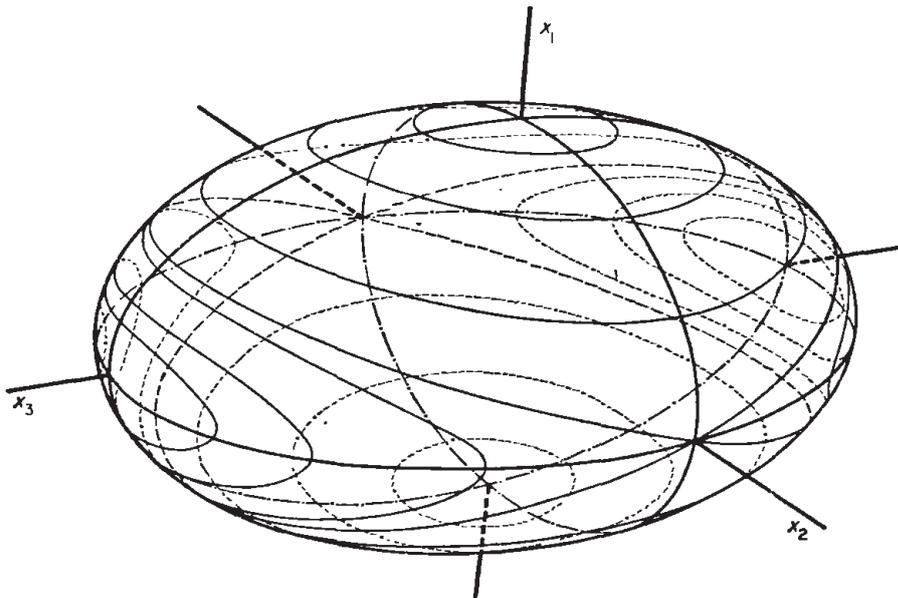


# Orienting pseudovectors and polhodes

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**Abstract.** We demonstrate that pseudovectors possess a “chirality” which is reversed by an improper rotation. The reversal of chirality of a pseudovector ought not be confused with reversing its direction, as commonly and mistakenly advertised. Such confusion is a source of repeated, drastic errors which must be eliminated before the dynamics of rigid body motion is investigated for algorithmic implementation.

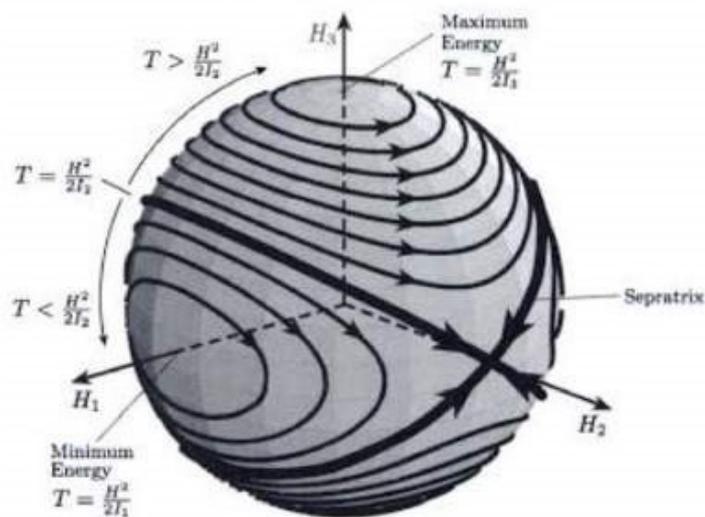
The well-established term *polhode* refers to the trajectory of the “tip” of the angular velocity in body’s frame. We shall refer to the trajectory of the “tip” of the angular momentum in body’s frame as the *momentum polhode*. The momentum polhodes were represented in Fig. 51 on p.117 of [1].



Pic. 1: The momentum polhodes as represented in Fig. 51 on p.117 of [1].

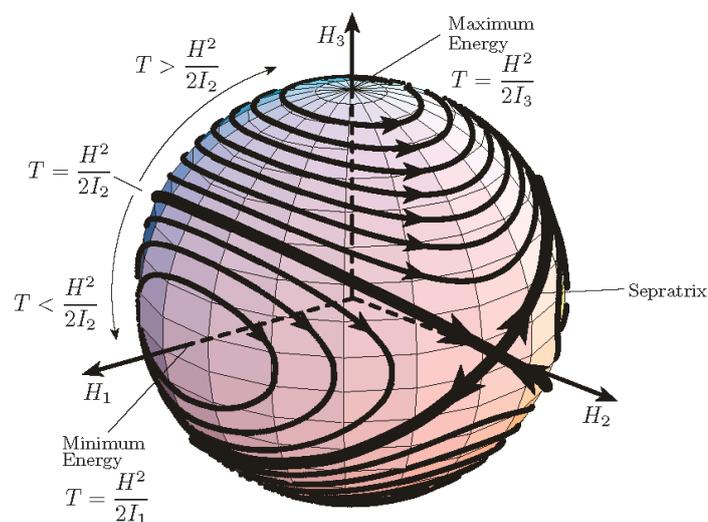
### A lasting “struggle” for orienting the momentum polhodes

The momentum polhodes were oriented in Fig. 4.7 on p. 147 of [2].



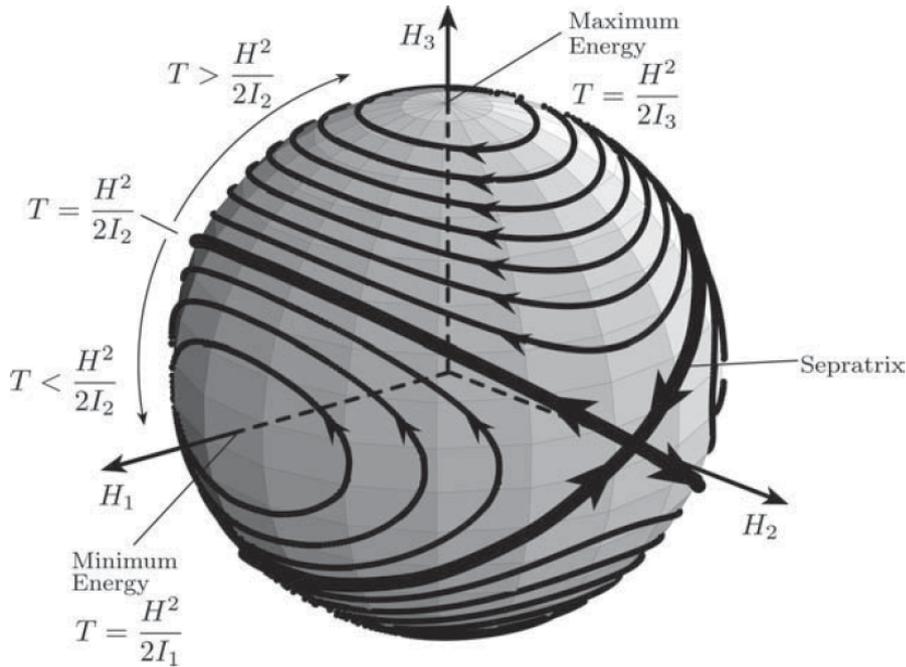
Pic. 2: The orientation of the momentum polhodes, as represented in Fig. 4.7 on p. 147 of [2].

The orientation of the separatrices was subsequently corrected in [3]. That correction was duplicated in [4]. The same correction was suggested by F.G. Uskov (Skolkovo Institute of Science and Technology) in a private correspondence.



Pic. 3: The orientation of the momentum polhodes as corrected in [3, 4].

Later yet, another orientation reversing correction was presented in Fig. 4.10 on p. 167 of [5].



Pic. 4: The orientation of the momentum polhodes, as further corrected in Fig. 4.10 on p. 167 of [5].

The latter correction was followed by another correction of the orientation of the separatrices in Fig. 4.9(b) of [5], which, according to [6], must be reversed.

### Is the “struggle” for orienting the momentum polhodes over?

Regardless of the adopted correction (in either Pic. 3 or Pic. 4) we might observe that the four separating momentum polhodes are now divided into two pairs, each pair lies on a single circle. Now the two separating momentum polhodes on a circle “originate” from “a critical point” and “terminate” in an “opposite” critical point. Yet, one must note here that the orientation (whether outwards or inwards) of a non separating polhode in either the “original” picture (Pic. 2) or in its “correction” (Pic. 3 and Pic. 4) is not preserved after subjecting it to “central symmetry”. In other words, the orientation of the path of the “tip” of the angular momentum opposes the orientation of the path of its “tail”. The case of a separating momentum polhode is exceptional since both the “tip” and the “tail” lie on the same circle, “carrying” a pair of separating momentum polhodes with two distinct (opposing to each other) orientations, yet originating (and terminating) in a common point.

On the other hand, we must also note that any separating polhode (regardless of its orientation) does “separate” momentum polhodes, oriented outwards, from momentum polhodes, oriented inwards.

Thus, the orientation of the momentum polhodes (which determination was laudably and tenaciously pursued in [2, 3, 4, 5, 6]) turned out (after all) being independent of the chosen orientation of body’s coordinate system (whether it was right-handed or left-handed)! We must, therefore, admit that no orientation of momentum polhodes is preferred, as no time direction is preferred (as far as the equations of rigid body motion are concerned). The authors of [1] might have (rightfully) suspected that they could not unambiguously orient the momentum polhodes (in Pic. 1), so they avoided discussing this subtle issue (which was commendably addressed by others).

## Conclusion

A source of widespread confusion is the conventional view of a pseudovector, “requiring” it to reverse its direction if subjected to an improper rotation. Such common view would preclude us from distinguishing a pseudovector from its image, if obtained via a central symmetry transformation. However, as we already clarified in [7, 8, 9, 10], a pseudovector is a “chiral” object and a momentum polhode which trace the “tip” of the angular momentum better be distinguished from its “mirror” image. A right-handed coordinate system, a right-handed pseudovector and a right-handed momentum polhode are “mirror” reflected to a left-handed coordinate system, a left-handed pseudovector and a left-handed momentum polhode, respectively.

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