Symbolic Sequences in the Analysis of Trajectories of Triple Black Holes

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Abstract. We analyse the orbits of triple black holes. Symbolic sequences based on binary approaches are constructed by numerical integration of the equations of motion. The sliding window method is used to detect periods of active triple interactions and long ejections.

Introduction

It is thought that at the center of most massive galaxies there exists a supermassive black hole [1]. As galaxies merge - and we now have evidence that they do [2], these supermassive black holes are brought close enough together for interaction between them to occur. When a third galaxy merges with these two there is then the formation of a triple system of supermassive black holes. Triple systems of smaller mass bodies like stellar black holes and stars have been known to exist [3]. The question of how these triple systems evolve is an area of great focus as we enter the era of gravitational wave physics. These multiple black hole systems provide a rich source of black hole mergers which existing and upcoming gravitational wave detectors like LIGO and LISA can easily detect.

In work that is in preparation for publication by the authors [4], Burrau's problem of three bodies [5] was studied in depth as an extension of the work conducted by Valtonen et al. in 1995 [6]. Black holes were placed at the vertices of Pythagorean triangles with their mass units reflecting the length of the sides of the triangle. Sixteen Pythagorean triangles were studied - those with hypotenuse less than 100. Numerical integration of orbits was conducted using ARCcode by Mikkola et al. [7]. It was found that there was strong correlation between the mass unit and the mergers occurring within a system. The lifetime of the systems was found to decay exponentially as the mass unit of the systems was increased.

Another parameter that was of interest was the number of binary encounters within a system's lifetime. This was used as a descriptor of how interactive a

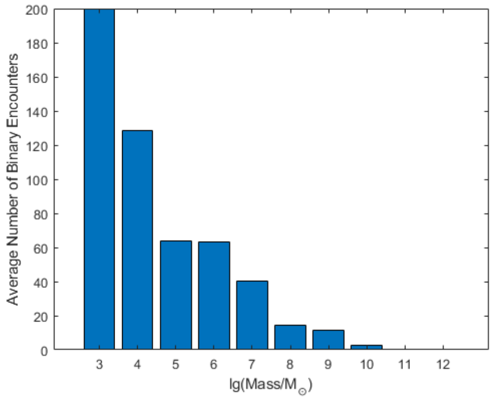

FIGURE 1. The average number of binary encounters per mass unit for mass unit range of 10^3M$_\odot$ to 10^{12}M$_\odot$.

system is. At every time step, the distances between the bodies are checked and when there is a local minimum value, a binary encounter is detected.

It was expected that as the mass of the systems increase that the number of binary encounters would decrease. This was corroborated for the most part by the results of our simulations (See Figure 1), however, there were outliers with large numbers of binary encounters in some large mass cases. In this work presented here, we look closely at these simulations that were considered outliers within their mass unit range.

Upon checking the trajectory of these systems it was concluded that at certain points one of the bodies would experience a long ejection. During this time the remaining two bodies would form a temporary binary pair which accounts for a large number of binary encounters being detected. In some cases this ejection and binary pair formation happened multiple times with bodies interchanging with each other. Using symbolic sequences (outlined in the following section) helps to easily identify systems that have high numbers of binary encounters due to triple interactions vs. systems that simply have long ejections.

Analysis of Symbolic Sequences

To analyse systems with large numbers of binary interactions, the sliding window method is used: we choose the window size, select the sub-sequence of this size starting from the beginning and calculate Shannon entropy. We then move one position to the right and repeat the process.

A plot of received entropies for the 48,55,73 triangular configuration (with mass unit 10^6M$_\odot$ and distance unit 1 parsec) is shown in Figure 2. Regions of

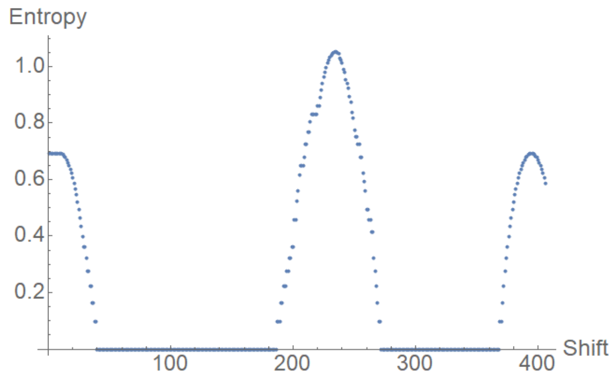

FIGURE 2. Received Entropies. Non-zero regions are where there is active triple interplay while zero regions are long ejections.

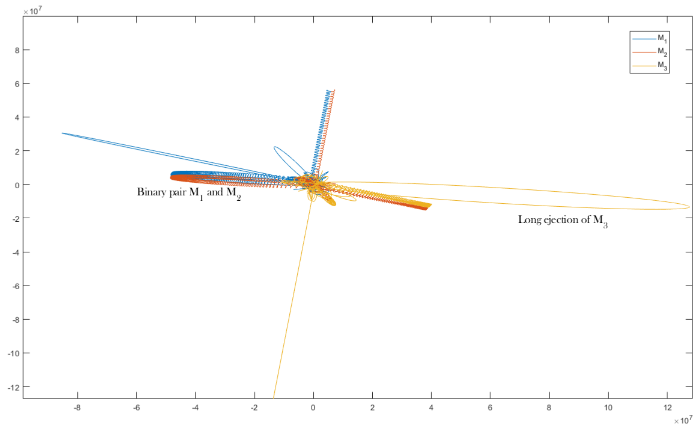

FIGURE 3. The orbital plot for the 48,55,73 triangular configuration with mass unit $10^6 M_\odot$ and distance unit 1 parsec. Temporary binary pair formation during a long ejection can be seen multiple times and one such event is labelled.

zero entropy are due to two long ejections while period of active triple interplay corresponds to the region with high value of the entropy in between.

The corresponding orbital plot of this system is shown in Figure 3. The temporary binary pairs are easily distinguishable in the plot. These correspond to the third body being ejected for a long duration. Entropy is zero during these events. The plot also shows chaotic triple interaction when all three bodies are within the vicinity of each other (at the center of the plot).

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