Photo illustration by Lucy Reading-Ikkanda/Quanta Magazine.

Symbolic Sequences in the Analysis of Trajectories of Triple Black Holes

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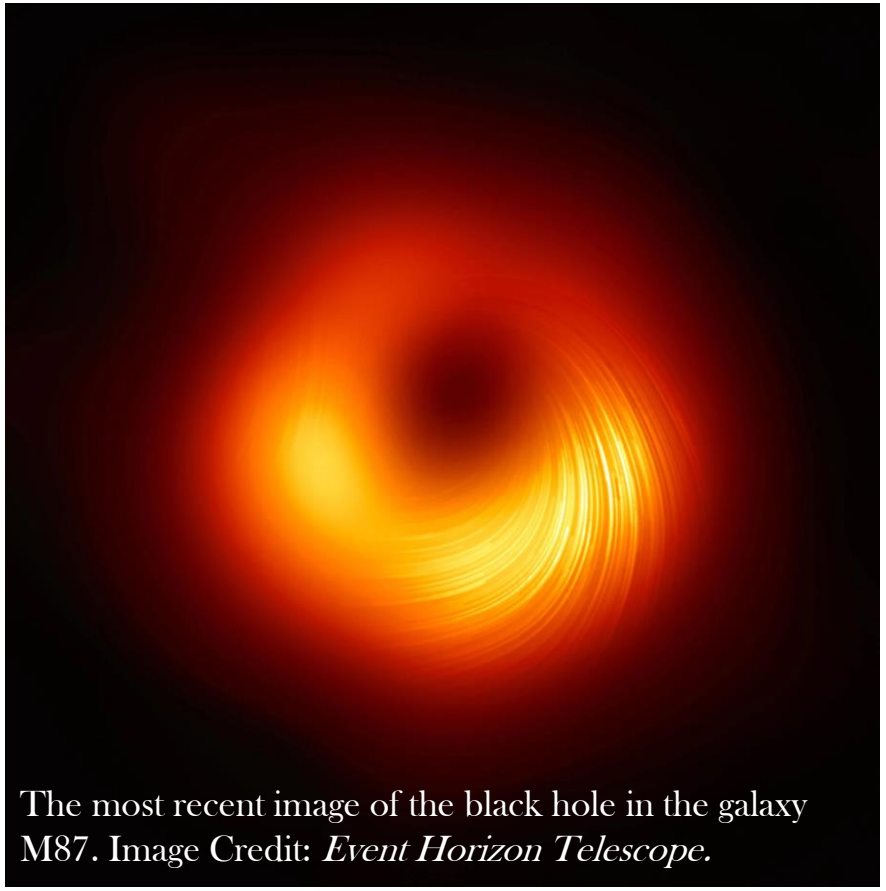
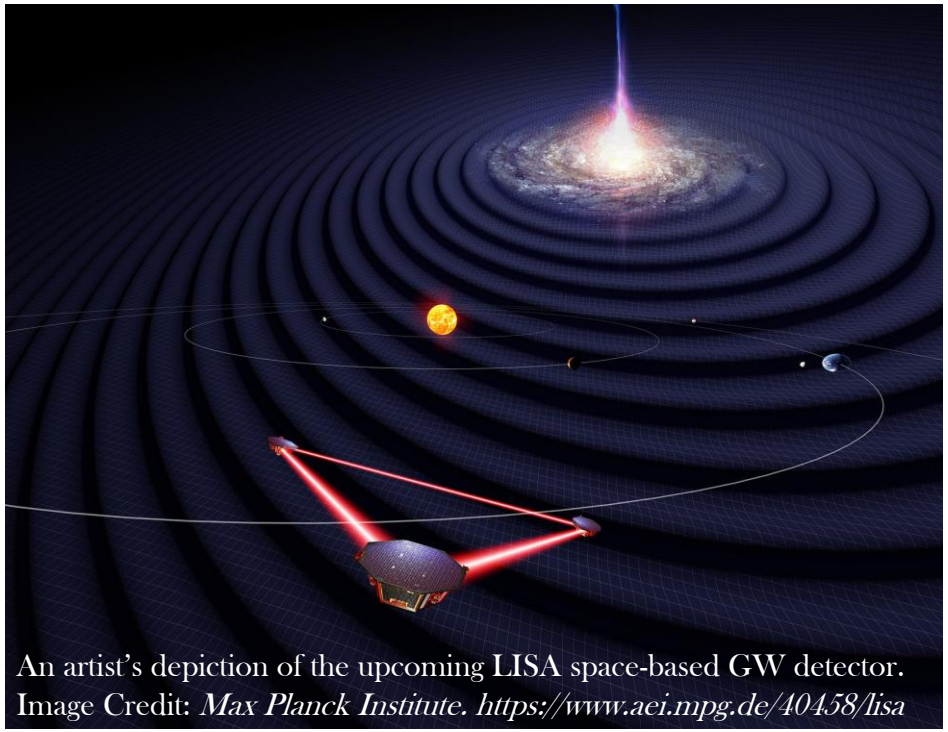
Black Holes are thought to be abundant throughout the universe.



The most recent image of the black hole in the galaxy M87. Image Credit: *Event Horizon Telescope.*

We are becoming equipped with technology to detect and even image black holes.



An artist's depiction of the upcoming LISA space-based GW detector. Image Credit: *Max Planck Institute. https://www.aei.mpg.de/40458/lisa*

Triple Black Hole Systems have been observed.

NGC 6240 is an example of a merger of three galaxies.

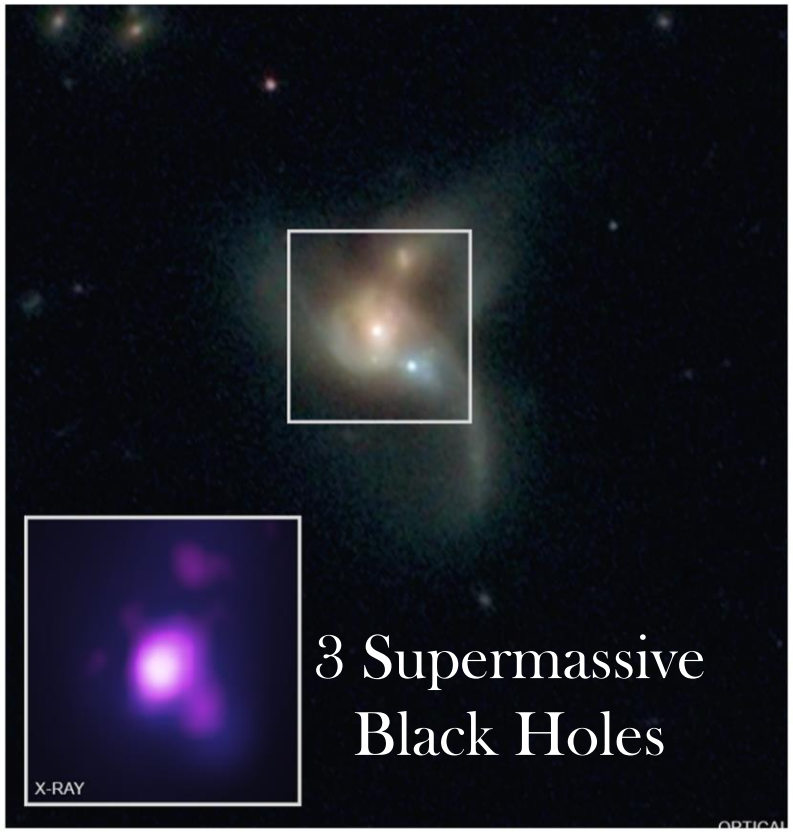
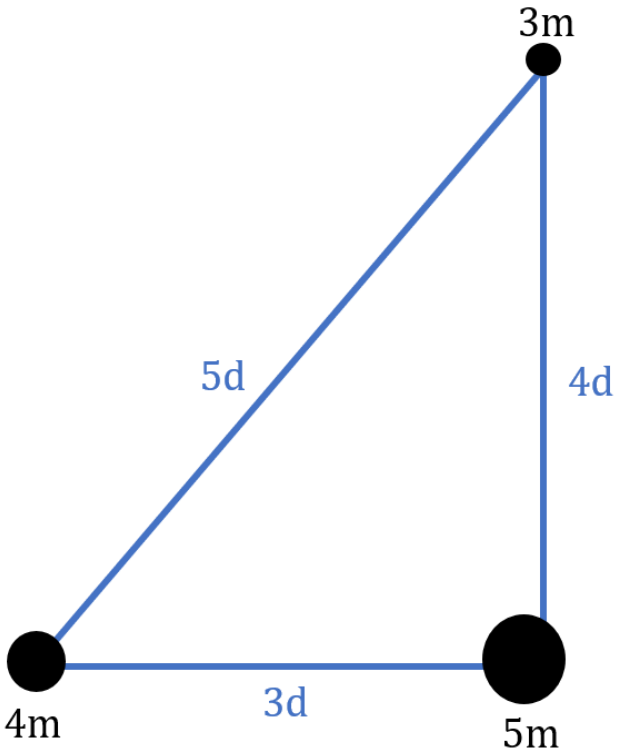


Image Credit: *NASA Chandra. https://www.nasa.gov/mission_pages/chandra/images/found-three-black-holes-on-collision-course.html*

Burrau's Problem of Three Bodies [1]



The classical 3,4,5 system studied by Burrau (1913).

Generalised by Valtonen et al. in 1995 [2]

Burrau's three-body problem in the post-Newtonian approximation

M. J. Valtonen, S. Mikkola and H. Pietilä
Turku University Observatory, University of Turku, Tuorla, FIN-21500 Piikkiö, Finland

Redone in 1995 with the aid of computers with correction terms up to the 2.5^{th} order

We extended their study where we focus on the effect of mass [4]. The following parameters were studied:

1. The number of mergers (we found as mass increased so did the number of mergers)
2. The lifetimes of the systems (we found that as mass increased the lifetime decreased exponentially)
3. The number of binary encounters in each system

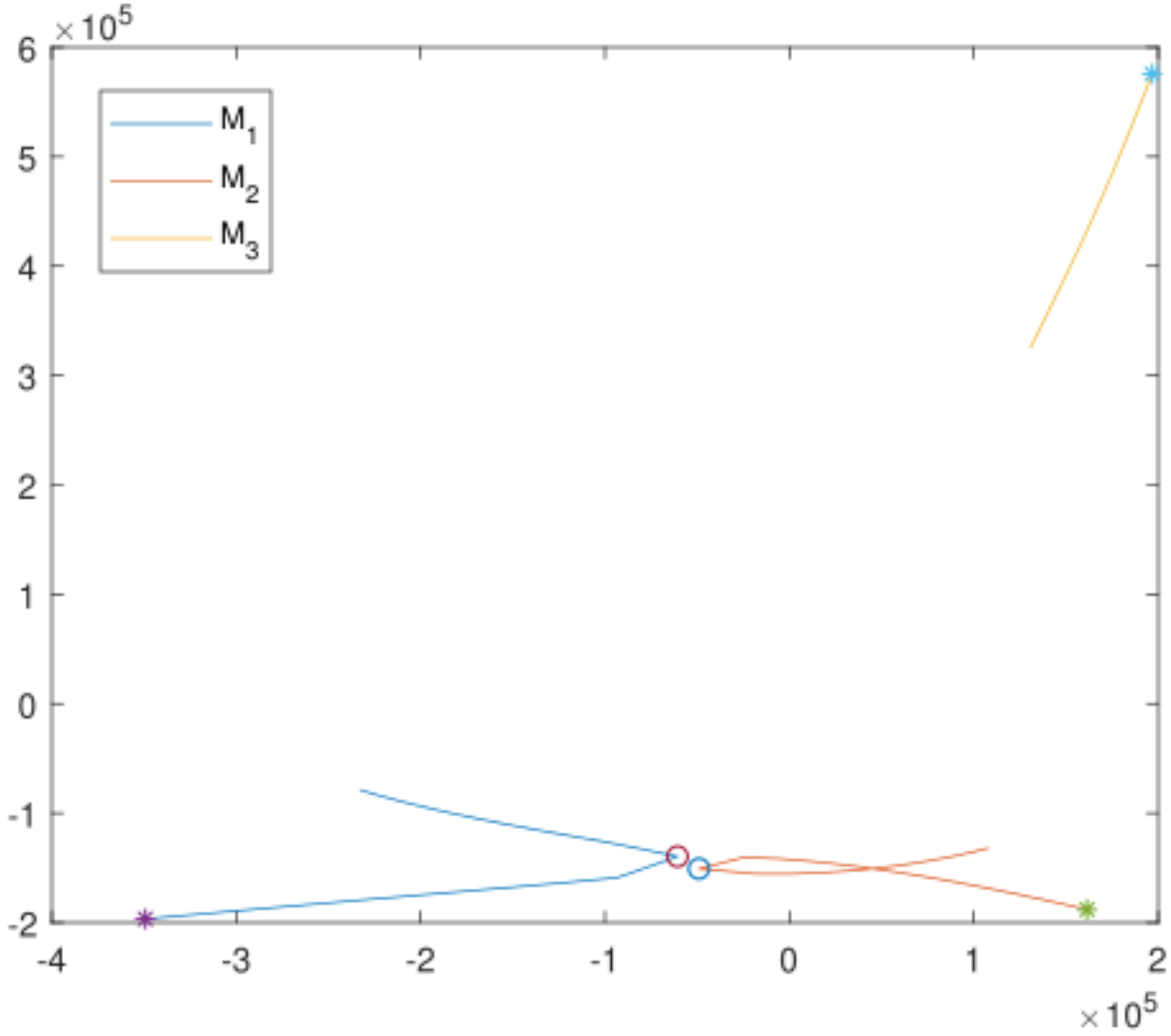
Simulations were done using code by Mikkola et al. [3].

Simulations were conducted using 2.5^{th} order post-Newtonian equations. The code used has been updated from the one in 1995.

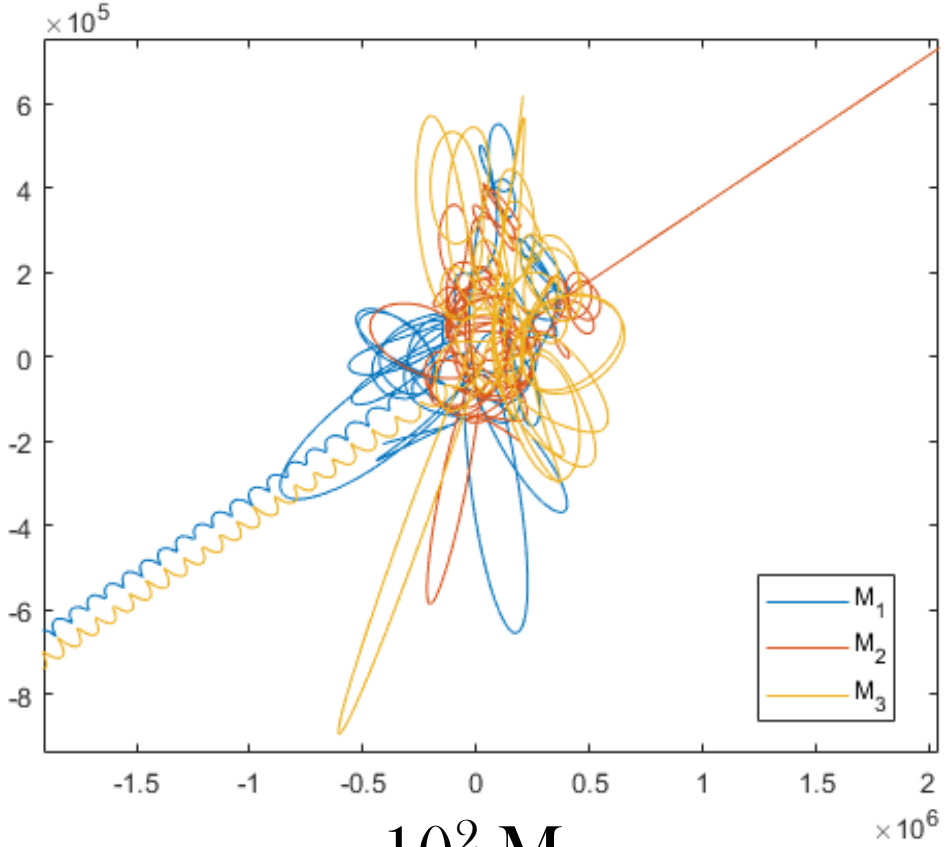
Binary Encounters

At each timestep, the distances between bodies are checked. When there is a local minimum, a binary encounter is detected.

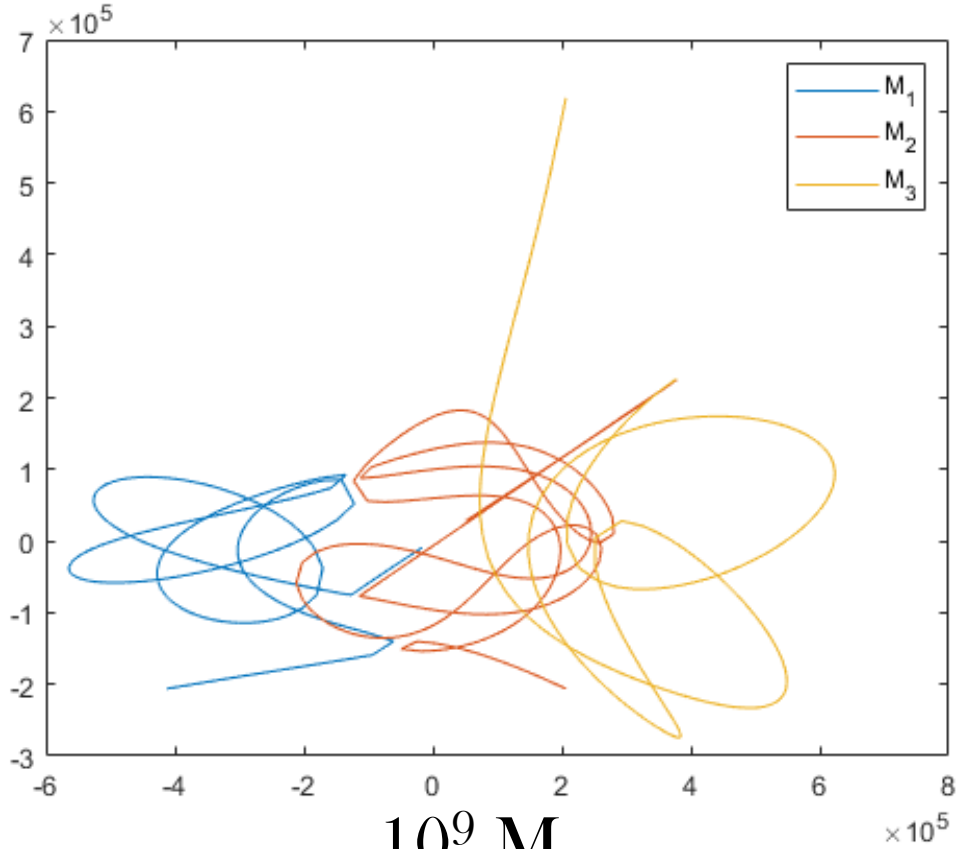
We use this to describe how interactive systems are.



Small mass systems tend to have more binary encounters.

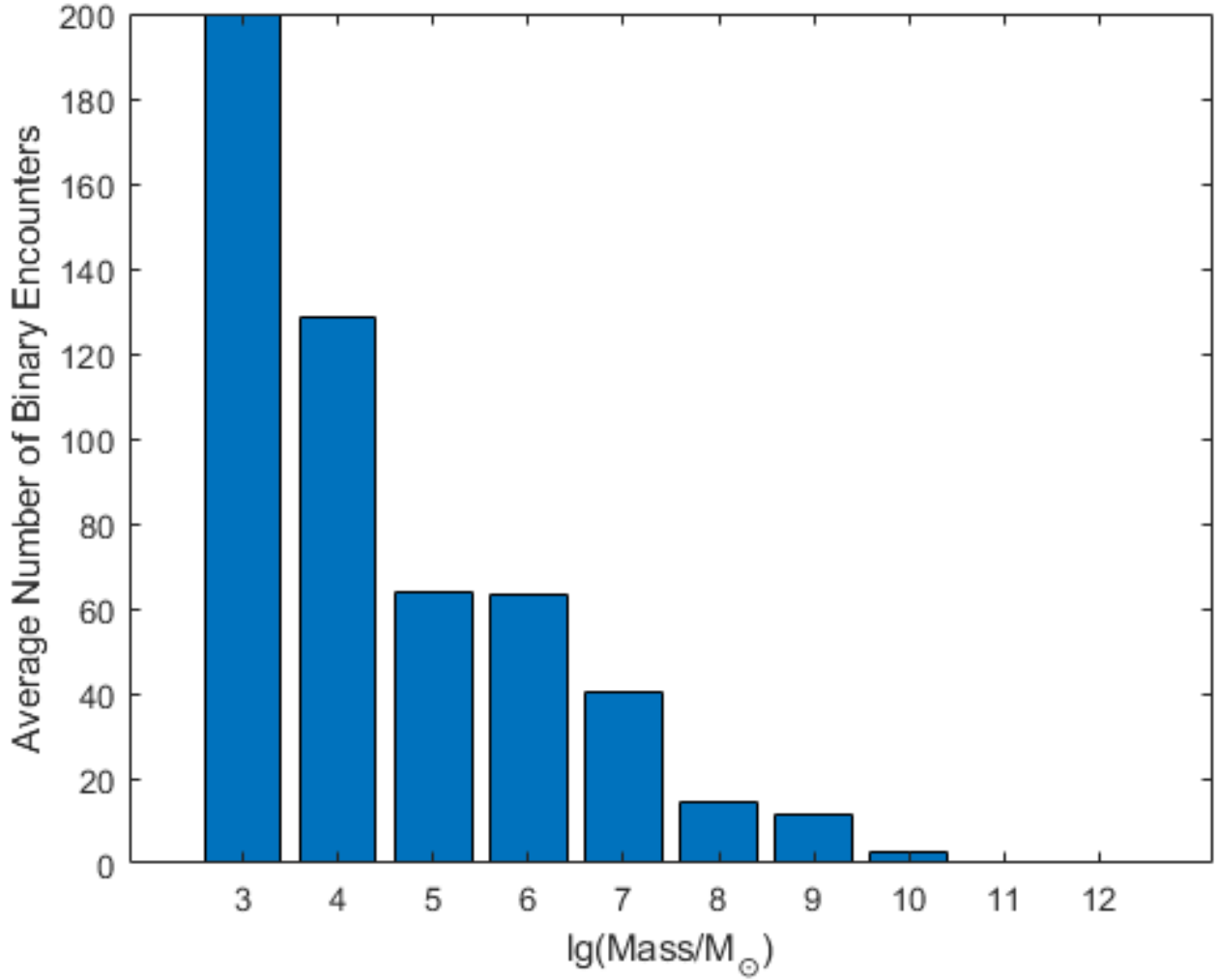


$10^2\ M_\odot$



$10^9\ M_\odot$

Average Binary Encounters vs. Mass - 10^3 $M_\odot$ - 10^{12} $M_\odot$

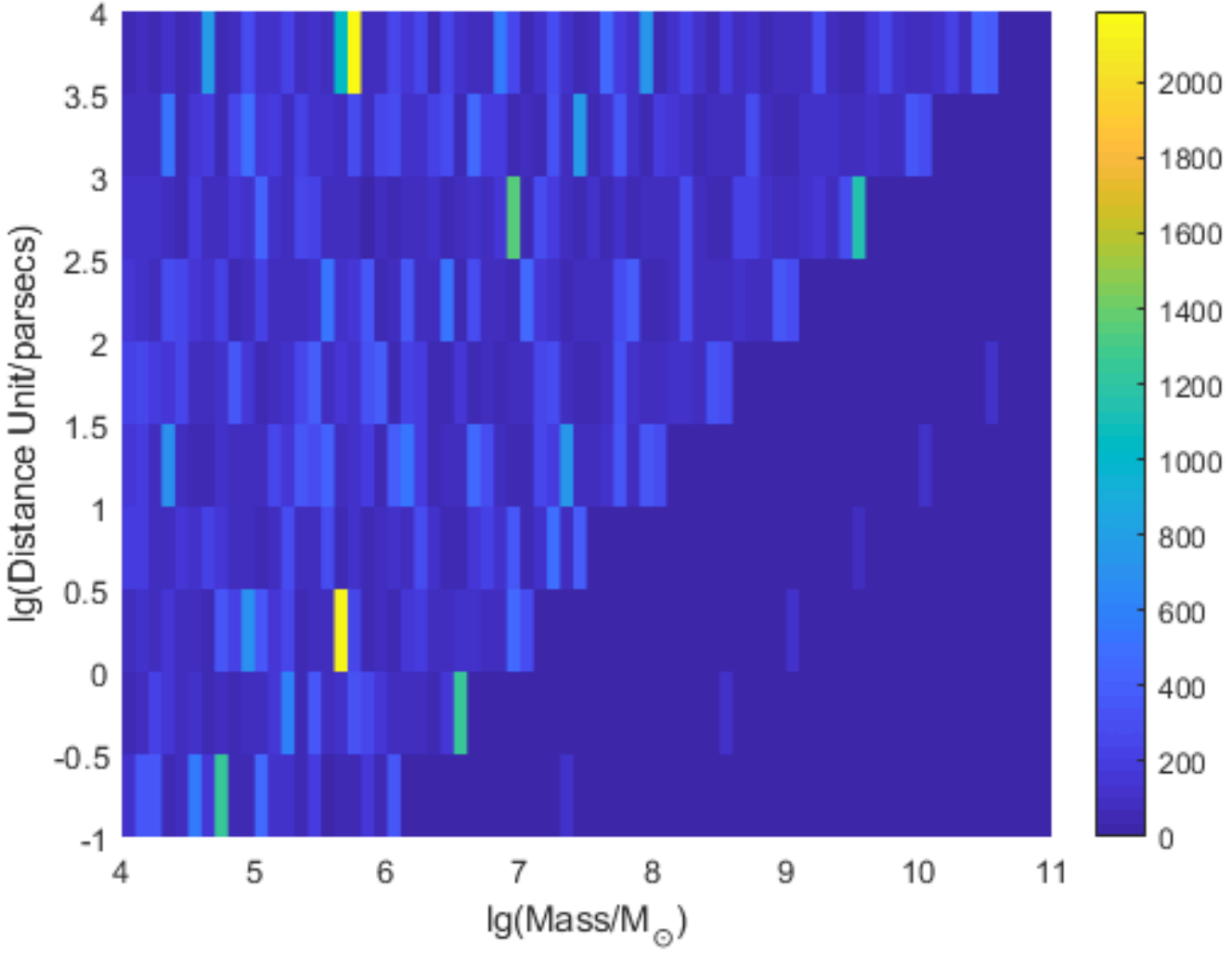


There were obvious outliers when studying the 3,4,5 system.

The number of binary encounters as both mass and distance are increased for the 3,4,5 triangle.

Here mass unit increases - 10^4 $M_\odot$ - 10^{11} $M_\odot$ and distance unit – 10^{-1}pc – 10^4pc.

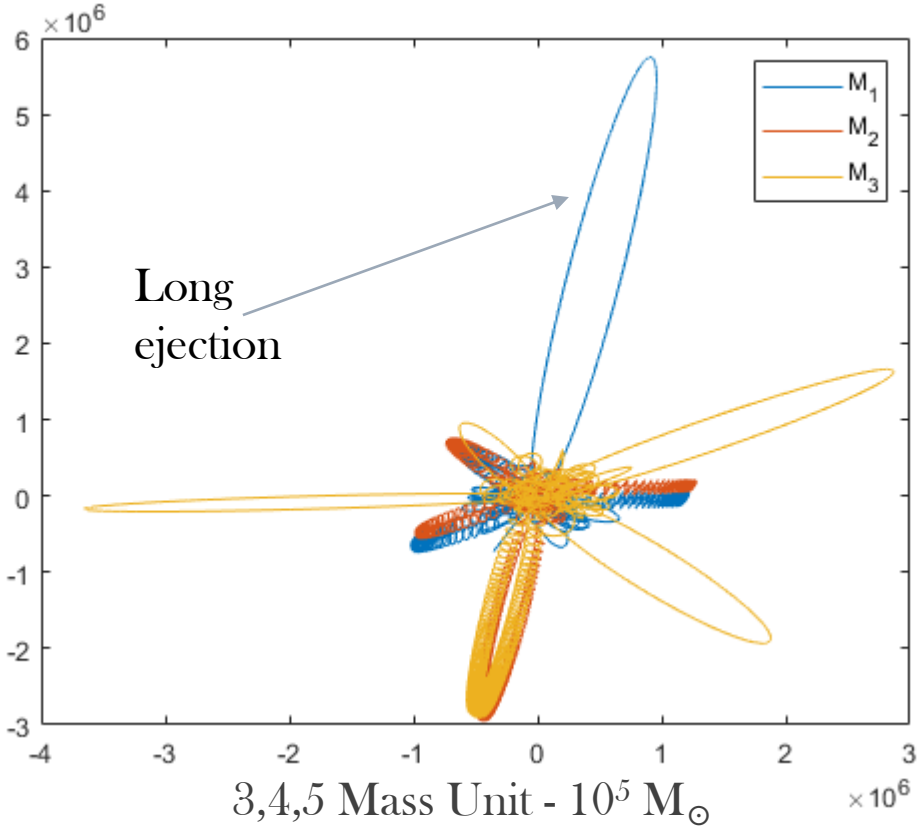
The Number of Binary Encounters



However, there were some outliers...

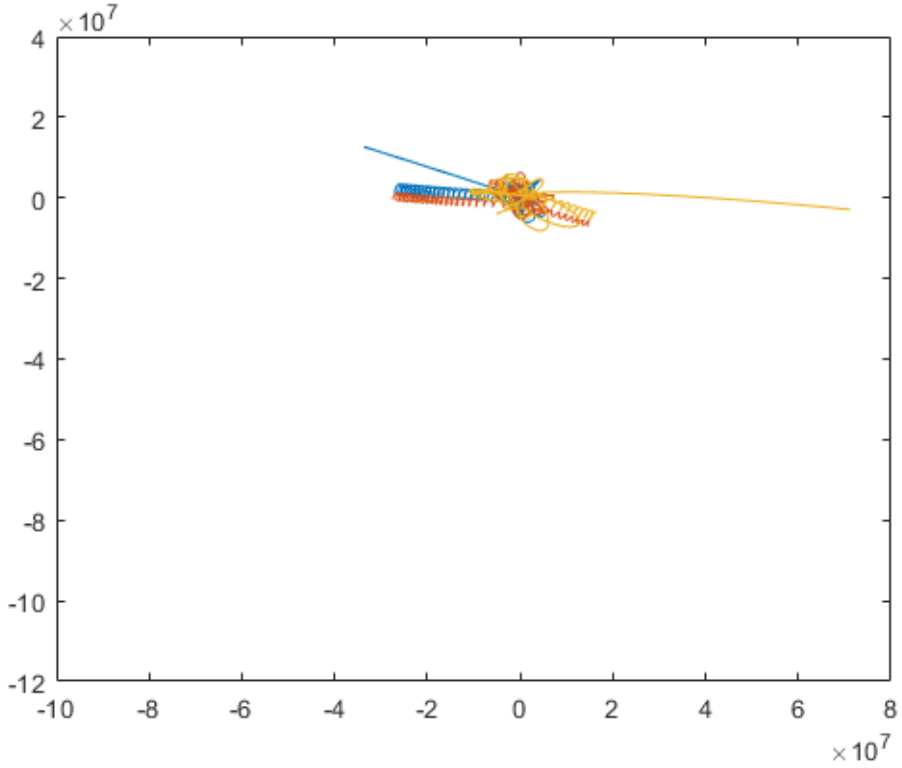
For example, the (48,55,73) triangle with a mass unit of 10^6 $M_\odot$ had 456 binary encounters. The average for this mass unit is around 60.

Long ejections/third body far away or very interactive?

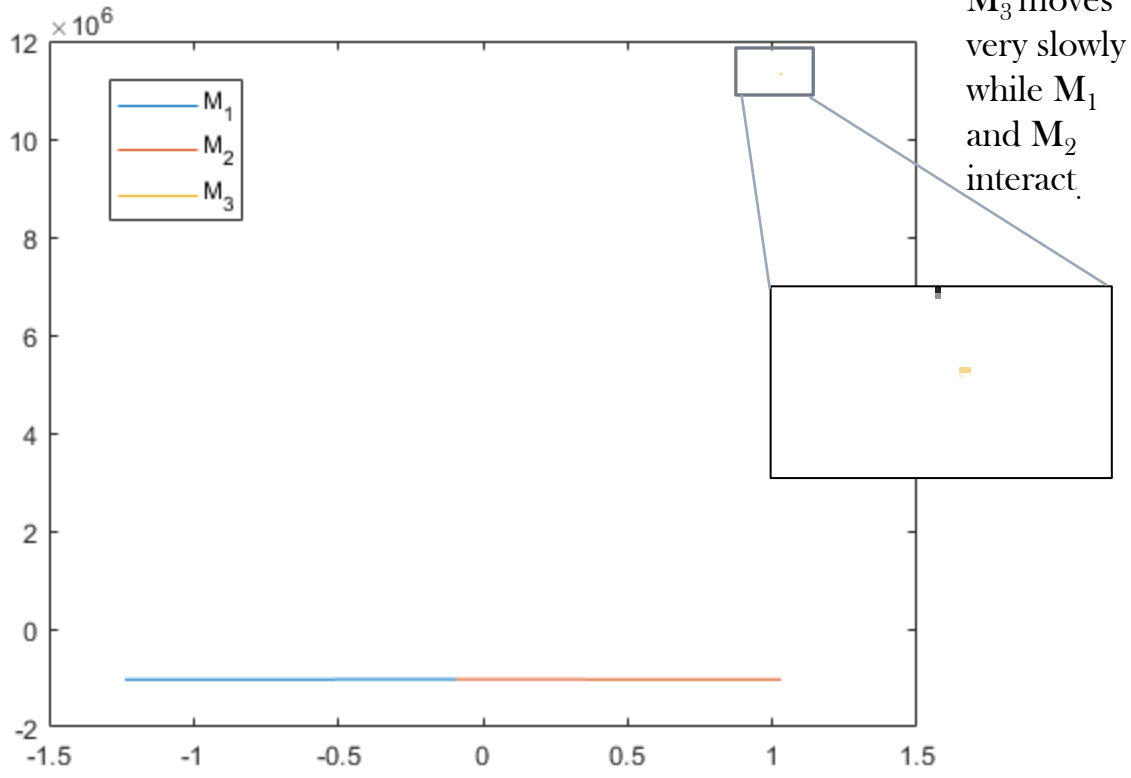


3,4,5 Mass Unit - 10^5 $M_\odot$
- Binary Encounters = 348

48,55,73 Triangle – Long Ejection



Or maybe the third body is far away and there is not much triple interplay



M_3 moves very slowly while M_1 and M_2 interact.

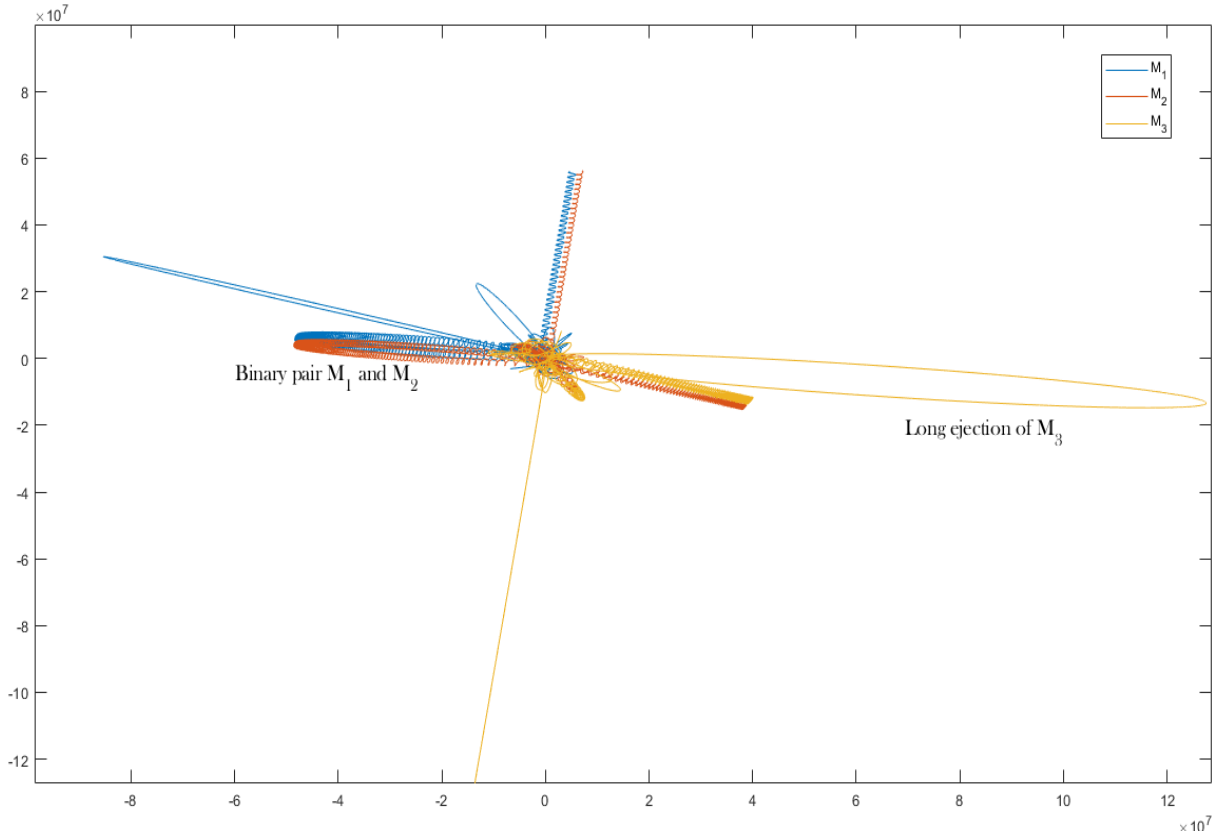
11,60,61 Mass Unit - 10^4 $M_\odot$
- Binary Encounters = 564

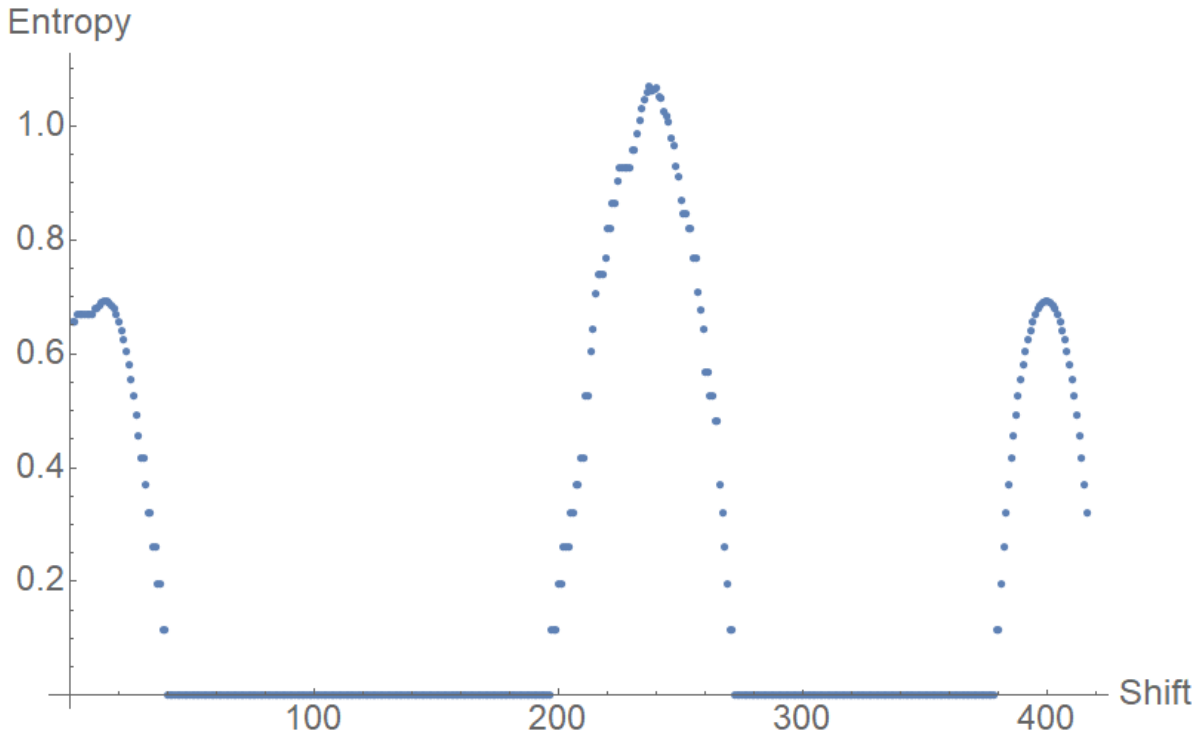
Symbolic Sequences

Using symbolic sequencing of the binary encounters to plot Shannon entropy helps to differentiate between these two types of systems.

Refer to Prof. Mylläri's talk - *Free-fall three-body problem and complexity of finite symbolic sequences.* [5]

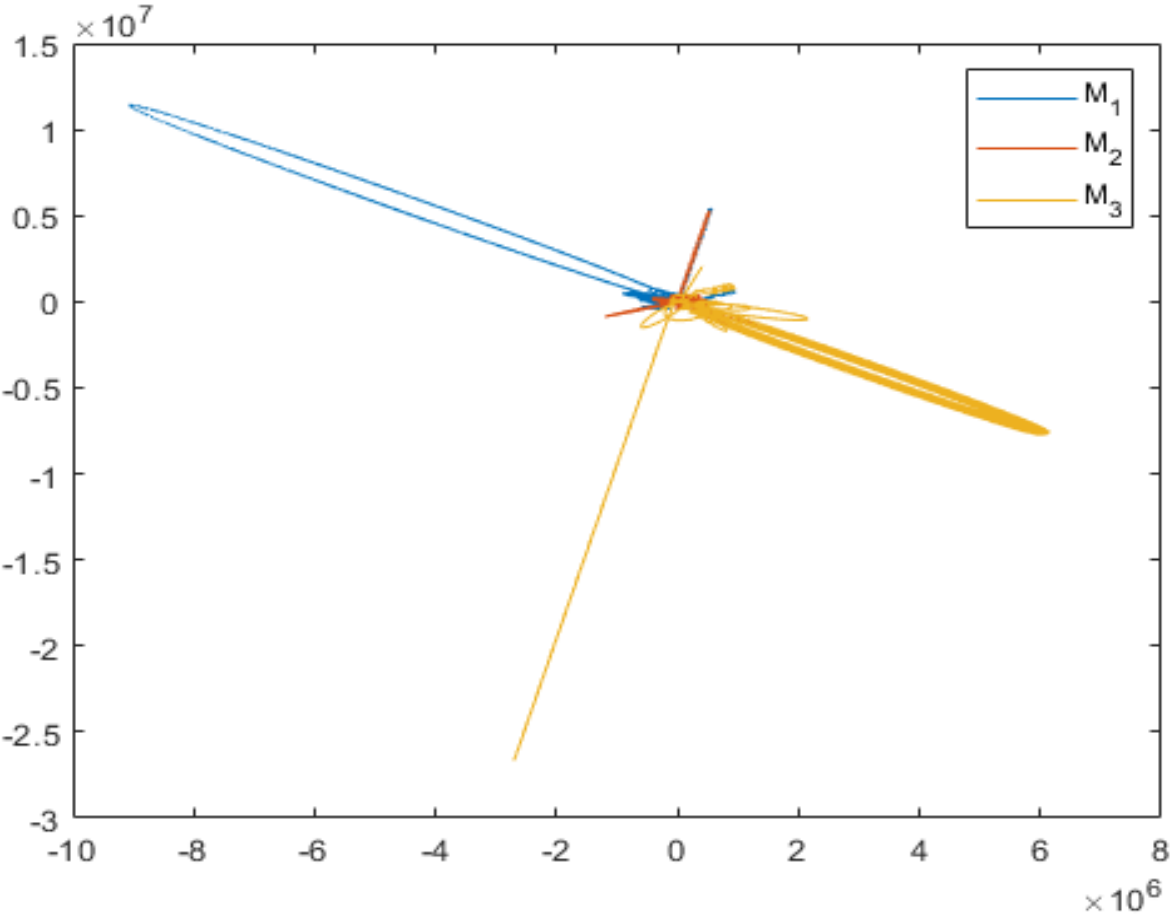
The sliding window method was discussed and is employed here as a practical example for analysing the symbolic sequences of binary encounters.

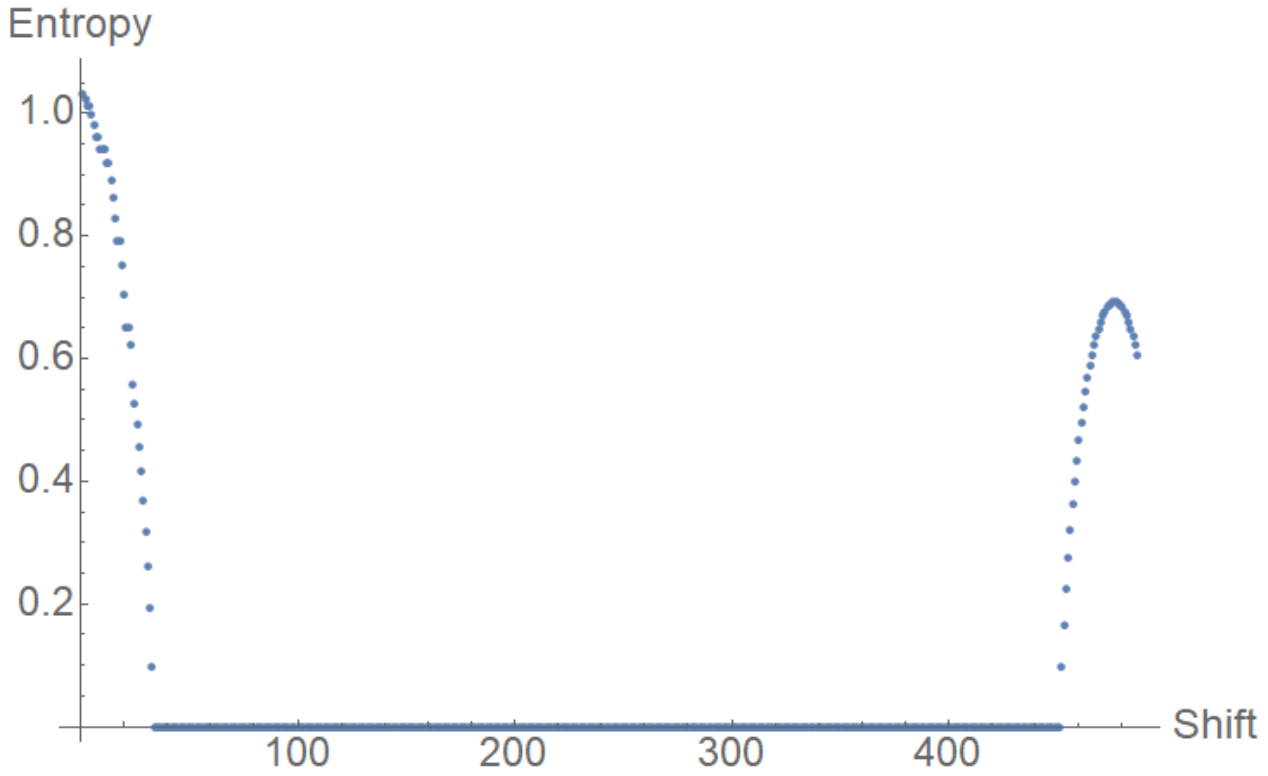




The (48,55,73) triangle - mass unit of 10^6 $M_\odot$

High entropy corresponds to triple interaction and interplay. Zero entropy corresponds to long ejections and binary encounters during this time between temporary binary pairs.





The (5,12,13) triangle - mass unit of 10^4 $M_\odot$

High entropy corresponds to triple interaction and interplay. Zero entropy corresponds to long ejections and binary encounters during this time between temporary binary pairs.

Conclusion:

We use the symbolic sequence of binary encounters to plot Shannon entropies. This lets us see which triples are more interactive and which have long ejections.

References:

[1] C. BURRAU, Numerische Berechnung eines Spezialfalles des Dreikörperproblems. Astronomische Nachrichten Volume 195 (Issue 6), 113–118 (1913).

[2] M. J. VALTONEN, S. MIKKOLA, H. PIETILÄ, Burrau's three-body problem in the post-Newtonian approximation. Monthly Notices of the Royal Astronomical Society Volume 273 (Issue 3), 751–754 (1995).

[3] S. MIKKOLA, K. TANIKAWA, Implementation of an efficient logarithmic-Hamiltonian three body code. New Astronomy Volume 20, 38–41 (2013).

[4] A. S. CHITAN, A. MYLLÄRI, S. HAQUE, Relativistic Effects on Triple Black Holes: Burrau's Problem Revisited. *Preprint*. 2020.

[5] A. MYLLÄRI , T. MYLLÄRI, N. VASILIEV, V. DUZHIN, Free-fall three-body problem and complexity of finite symbolic sequences, https://pca-pdmi.ru/2021/files/13/MMVD.pdf