Linearizability property of Lie symmetry algebra

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1 Abstract

An algebraic framework is proposed for determining if a scalar ordinary differential equation could be mapped into a linear one. We call this property linearizability or exact linearization. The problem is split into two parts: (I) obtain a certificate which ensures the existence of a linearizing mapping and (II) construct the determining system for it. The complexity bottleneck of this technique is the completion to a standard basis (Riquier basis of symmetry infinitesimals for problem (I)) and the Thomas Decomposition of the nonlinear determining system of the linearizing mapping for problem (II). In both cases, the numbers of dependent and independent variables remain fixed which defines an upper complexity bound of the corresponding algorithms. The main difference is the transformation of the infinitesimal symmetry generators by means of Bluman-Kumei equations. We prove a theorem on the construction of finite-dimensional linearizing systems, which is important in theory and application. Moreover, we illustrate our approach with several examples which admit exact linearization by point or contact transformations.