Classical Fisher information metric from the Wigner function of $N-$level system

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A long history of various efforts to extend the concepts and methods of classical information theory to the quantum world indicate existence of fundamental obstacles on this way. At the same time, searching for methods to overcome these, new aspects of a classical-quantum correspondence are revealed. A well-known example of this is the attempt to introduce quantum analogue of the so-called classical statistical manifold of probability distributions, which is widely exploited in the classical information geometry [1]. The studies of the geometrical aspects of the quantum information are interesting not only from a pure mathematical standpoint but became very actual owing to the practical purposes. Due to a request coming from the quantum technology, formulation of the quantum estimation theory [2] turn to be in the frontier of a modern research. Particularly, the issue of interrelations between the phase space quasidistributions and classical Fisher metric are of current interest. Our studies are devoted to this issue and in the report we claim a representation of the classical Fisher metric corresponding to a quantum system in states admitting description in terms of a positive definite Wigner function.

The derivation of the above mentioned representation for the Fisher information matrix is based on the recently elaborated approach to phase-space description of $N-$dimensional quantum system using the Wigner function defined over the flag manifold $\Omega_N := U(N)/U(1)^N$ [3]. Let $\mathfrak{P}_N$ be a state space of a quantum system and π be a quotient (canonical) mapping corresponding to the adjoint $SU(N)$ group action:

$$\pi : \ \mathfrak{P}_N \longrightarrow \mathcal{O}[\mathfrak{P}_N] = \mathfrak{P}_N/SU(N)\,, \tag{1}$$

and $\Omega_N^{(+)}[\varrho]$ denotes a subset of phase space Ω_N , where the Wigner function of a given state ϱ is non-negative:

$$\Omega_N^{(+)}[\varrho] = \{\, x \in \Omega_N \ \mid W_\varrho(\Omega_N) \geq 0 \,\}\,. \tag{2}$$

Then one can consider the states $\varrho \in \mathfrak{P}_N^{(+)}$ such that every $\Omega_N^{(+)}[\varrho]$ coincides with the whole phase-space:

$$\mathfrak{P}_N^{(+)} = \{\varrho \in \mathfrak{P}_N \ \mid \Omega_N^{(+)}[\varrho] = \Omega_N \, \} \,. \tag{3}$$

We will show that this subset is not empty and moreover the set $\mathcal{O}[\mathfrak{P}_N^{(+)}]$, defined as the image of $\mathfrak{P}_N^{(+)}$ under the quotient mapping π ,

$$\mathcal{O}[\mathfrak{P}_N^{(+)}] = \{\pi(x) \ \mid x \in \mathfrak{P}_N^{(+)} \, \} \,, \tag{4}$$

represents the cone on the orbit space $\mathcal{O}[\mathfrak{P}_N]$. Hence, the Wigner functions of states from $\mathfrak{P}_N^{(+)}$ are proper positive definite probability distributions. Having this result one can compute the classical Fisher information associated to the probability distribution of states $\varrho \in \mathfrak{P}_N^{(+)}$ in terms of the corresponding Wigner function.

The derived representation for a classical Fisher matrix is exemplified for the case of qubit $(N = 2)$ and qutrit $(N = 3)$.

References

[1] Shun-ichi Amari, Information Geometry and Its Applications, Springer Japan 2016

[2] M.G.A Paris, Quantum estimation for quantum technology, Int. J. Quant. Inf. *7*, 125 (2009).

[3] V. Abgaryan and A. Khvedelidze, *On families of Wigner functions for N-level quantum systems*, *https://arxiv.org/pdf/1708.05981.pdf* (2018).

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