

Hierarchy of classicality indicators of N -level systems

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The representation of finite-dimensional quantum systems in a phase space [1] inevitably leads to the problem of negativity of probability distributions defined over the phase space [2]. It is commonly accepted that this negativity is an essential attribute of “quantumness” of a system and therefore can be used for evaluation of the quantitative characteristics of states [3]. Following this idea and basing on the algebraic method of construction of the Wigner functions of N -level quantum systems [4, 5], we introduce the global indicator of classicality \mathcal{Q}_N [6, 7, 8] defined as a relative volume of a subspace $\mathfrak{P}_N^{(+)} \subset \mathfrak{P}_N$ of the state space \mathfrak{P}_N , where the Wigner quasiprobability distribution is positive. In the present report we analyse a refined hierarchy of measures of classicality corresponding to a natural *stratification of state space \mathfrak{P}_N by the unitary orbit types*. The adjoint action of $SU(N)$ group on density matrices $\varrho \in \mathfrak{P}_N$,

$$g \cdot \varrho = g\varrho g^\dagger, \quad g \in SU(N), \quad (1)$$

induces the state space decomposition into the strata:

$$\mathfrak{P}_N = \bigcup_{\text{orbit types}} \mathfrak{P}_{[H_\alpha]}. \quad (2)$$

The components of decomposition (2) are determined by the *isotropy group* H_ϱ of a point $\varrho \in \mathfrak{P}_N$,

$$\mathfrak{P}_{[H_\alpha]} := \{\varrho \in \mathfrak{P}_N \mid H_\varrho \text{ is conjugate to } H_\alpha\}. \quad (3)$$

Having in mind the above stratification, it is natural to define *global indicator of classicality* $\mathcal{Q}_N[H_\alpha]$ of states over a given stratum as the ratio:

$$\mathcal{Q}_N[H_\alpha] = \frac{\text{Vol}(\mathfrak{P}_{[H_\alpha]}^{(+)})}{\text{Vol}(\mathfrak{P}_{[H_\alpha]})}, \quad (4)$$

where $\mathfrak{P}_{[H_\alpha]}^{(+)}$ is the subset of stratum $\mathfrak{P}_{[H_\alpha]}$ where the Wigner quasiprobability distribution is non-negative. In the definition (4) the volumes are evaluated with

respect to the Riemannian metric on $\mathfrak{P}_{[H_a]}$ induced by the stratification embedding. In order to exemplify the introduced indicator of classicality, we explicitly evaluate the rate of quantumness-classicality for low-dimensional systems, such as a qubit and a qutrit.

References

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