

Change and Solve

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Abstract. The change of representation often gives a key to the efficient solution of problems in both numeric and symbolic computation, or in the mixed symbolic-numeric framework.

Popular examples of this technique include but not limited to

- change of basis in polynomial computations,
- modular and evaluation homomorphisms (involving Chinese remaindering or evaluation and interpolation),
- integral representation for computing combinatorial sums,
- fast Fourier transform,
- implicitization of a problem, etc.

In this talk we will give a survey of several popular problems along with discussion of how and why the change of representation improves the theoretical and practical running-time complexity of the employed algorithms. We will consider problems related to the symbolic summation, high-precision evaluation of rapidly convergent series, evaluation of closed form expressions over the regular intervals, and discuss various approaches to their acceleration based on the change of representation [1] - [24].

Discussion will be accompanied by demonstration of software implementations in Maple, Java and C programming languages.

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