Three-body dynamics: Agekian-Anosova region D

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Abstract. We discuss Agekian-Anosova homology region D and its impact on the studies of three-body dynamics.

Since it is impossible to obtain a general analytical solution to the three-body problem, the researchers are left with a numerical experiment. For the numerical integration of the three-body problem, it is necessary to set up the initial conditions: masses of the bodies, their coordinates and velocities. We need to choose for each body mass, 3 spatial coordinates and 3 initial velocities, 21 parameters in total. We can slightly reduce this number using the integrals of motion, but still the space of initial conditions will have a high dimension, which makes it difficult to choose the initial conditions and limits the ability to present the research results in a visual and easy-to-analyze form.

One can simplify the problem by considering all bodies of the same mass. Next simplification is to choose zero initial velocities. This simplifies the problem - from the spatial the problem becomes planar. In this case, it is necessary to specify initial coordinates of the bodies, 3 pairs, six numbers total, but the dimensionality is still high and does not allow visualization easily. The problem was solved when the famous region D appeared [1, 2], often called Agekian-Anosova region D or Agekian-Anosova map. Later, A.D. Chernin proposed the name homology region D [3].

The idea of the region D is very simple: if we place two bodies at the points with coordinates (-0.5, 0) and (0.5, 0), then we get all possible different geometric configurations by placing the third body in the area bounded by (positive) coordinate axes and arc of the unit circle centered at (-0.5, 0), see Fig. 1.

Introduction of the homology region D allows sistematical study of the free-fall equal mass three-body problem and natural visualization. One can study and display, e.g., life-time of the systems, see Fig. 2, or search for initial conditions leading to two- and three-body collisions after first, second, etc. approach (Fig. 3), analyse complecsity of trajectories (Fig. 4) and so on.

Homology region D can also be used as a map [4]: at any moment of time to analyze geometric configuration we can project the system into region D and

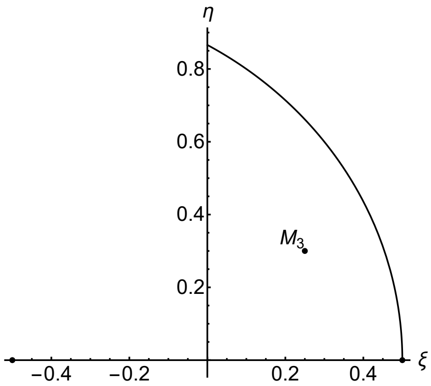

FIGURE 1. Agekian-Anosova Region D.

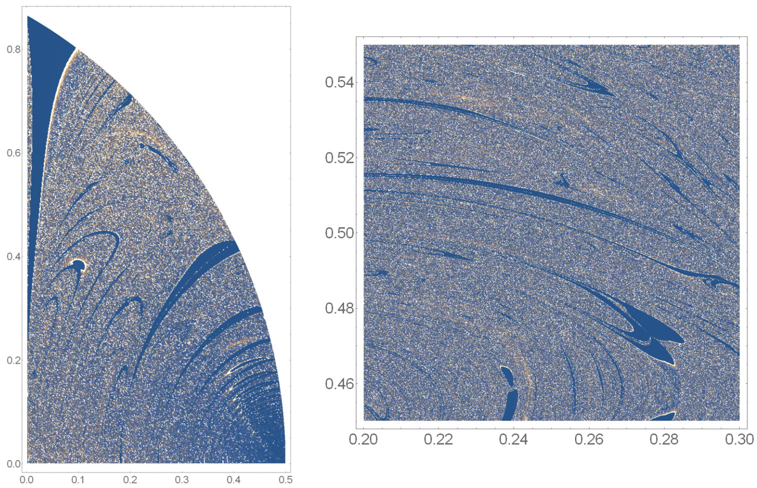

FIGURE 2. Left: Life time of three-body systems. Blue (dark) color correspond to short-living systems. Right: zoom into selected area.

follow the motion of the point representing the system. It can be done also in the case of non-planar (3D) motion. Since typical final stage of the evolution of three-body system with zero angular momentum is ejection, the point (0.5, 0) will play a role of attractor. Region D can be generalized to the case of different masses - in this case one would need to consider all six possible permutations of the bodies at the initial moment.

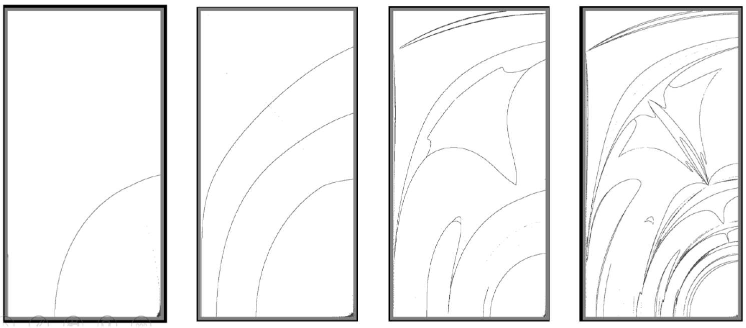

FIGURE 3. Two-body collision curves after first, second, etc. approach. Triple collisions can be revealed as intersections of the curves.

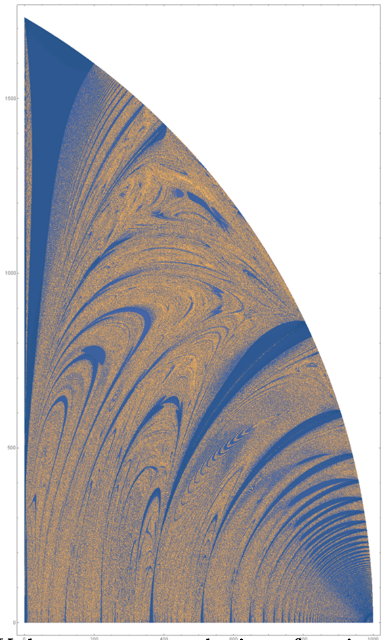

FIGURE 4. Kolmogorov complexity of trajectories. For each trajectory symbolic sequence was constructed. Complexity is estimated as a length of the archived sequence.

References

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