Around concurrent normal conjecture

Alexandr Grebennikov and Gaiane Panina

Given a smooth convex body $K \in \mathbb{R}^n$, its normal to a point $p \in \partial K$ is a line passing through p and orthogonal to ∂K at the point p. It is conjectured that for any convex body $K \in \mathbb{R}^n$ there exists a point in the interior of K which is the intersection point of at least $2n$ normals from different points on the boundary of K. The concurrent normals conjecture trivially holds for $n = 2$. For $n = 3$ it was proven by Heil via geometrical methods and reproved by Pardon via topological methods. The case $n = 4$ was completed also by Pardon.

Recently Martinez-Maure proved for $n = 3, 4$ that (under mild conditions) almost every normal through a boundary point to a smooth convex body K passes arbitrarily close to the set of points lying on normals through at least six distinct points of ∂K. He used Minkowski differences of smooth convex bodies, that is, the *theory of hedgehogs*.

We give a very short proof of a slightly more general result: *for dimension $n \geq 3$, under mild conditions, almost every normal through a boundary point to a smooth convex body $K \in \mathbb{R}^n$ contains an intersection point of at least 6 normals from different points on the boundary of K.*

Our proof is based on the bifurcation theory and does not use hedgehogs.

Acknowledgments. G. Panina is supported by RFBR grant 20-01-00070A; A. Grebennikov is supported by Ministry of Science and Higher Education of the Russian Federation, agreement 075-15-2019-1619.

Alexandr Grebennikov
Department of Mathematics and Computer Sciences
Saint-Petersburg State University
Saint-Petersburg, Russia
e-mail: sagresash@yandex.ru

Gaiane Panina
Dept. name of organization
Saint-Petersburg Department of Steklov Mathematical Institute
Saint-Petersburg, Russia
e-mail: gaiane-panina@rambler.ru