On basic stratified structures in quantum information geometry

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Searching for non-trivial physical consequences of quantum theory, the knowledge of the mathematical structure of the set of quantum states can be a reliable guide. The state space $\mathfrak{P}_N$ of an N-level quantum system consists of $N \times N$ Hermitian, normalized semi-positive density matrices,

$$\mathfrak{P}_N = \{\, X \in M_N(\mathbb{C}) \,|\, X = X^\dagger\,,\ X \geq 0\,,\ \mathrm{Tr}\, X = 1\,\}\,.$$

During the last two decades, following the request coming from the advanced quantum technologies and quantum information science, the state space $\mathfrak{P}_N$ has been studied in various contexts, among them convex-geometric, topological, differential-geometric, etc. (see, e.g. reviews [1, 2, 3] and references therein.)

In the present report, we discuss some features of the underlying stratified structure of $\mathfrak{P}_N$. It will be outlined that among three admissible partitions of $\mathfrak{P}_N$, namely by the adjoint $SU(N)$ orbits, by the corresponding orbit types, or by the subsets of density matrices with fixed ranks, only the last decomposition determines the Whitney stratification. Based on this observation, we expand some results of our recent paper [4], devoted to the study of the Bures-Fisher metric for rank deficient states, which are non-maximal dimensional strata of the Whitney stratification. We comment on the existence of a generalized stratified metric on the whole state space.

The first author was supported from EU Regional Development Fund-Project No.CZ.02.1.01/0.0/0.0/16_019/0000766. The research of A.K and D.M was supported in part by the Collaborative grant "Bulgaria-JINR".

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