0-1 laws in asymptotic combinatorics and central markov measures for continuous graphs

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Schort description of the talk

A class of continuous graded graphs of the type of Gelfand-Zeitlin schemes is defined and the set of all ergodic central measures of discrete type on the path space of such graphs is described. The main method consists of checking the ergodicity ("0-1 law") of the limit of Lebesgue measures on finite-dimensional convex compacts. As an application of the proposed approach, a new proof of the theorem is given that the list of ergodic discrete unitary invariant measures on the set of infinite Hermitian matrices is exhausted by the so-called Wishart measures.

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- Definition of continouous graded graphs with convex set of paths;
- Spaces of the Gelfand-Cetlin type;
- Central measures of discrete type;
- Systems of co-transition probabilities;
- Ergodic method.

Some examples

- Cesaro graph;
- Random walks in Weyl chambers;
- Graph of Hermitian matrices and graph of spectra;

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- ПЕРЕМЕЖАЕМОСТЬ И МНОГОГРАННИК ГЕЛЬФАНДА ЦЕТЛИНА.
- РАЗЛИЧНЫЕ СПОСОБЫ ОПИСАНИЯ ИНВАРИАНТНЫХ (ЦЕНТРАЛЬНЫХ) МЕР.
- МАРКОВСКИЕ ЦЕПИ — ОБЛАСТИ ДОСТИЖИМОСТИ И КОДОСТИЖИМОСТИ.

0-1 laws in asymptotic combinatorics and central markov measures for continuous graph

- ЗАДАЧА О КОМБИНАТОРНЫХ ЗАКОНАХ "0-1" .
- СВЯЗЬ С ТЕОРИЕЙ ФИЛЬТРАЦИЙ
- ЛЕММА ОБ ПРЕДЕЛЬНОЙ СИГМА-АЛГЕБРЕ.
- УНИТАРНО-ИНВАРИАНТНЫЕ МЕРЫ НА ЭРМИТОВЫХ МАТРИЦАХ.
- МЕРЫ УИШАРТА.

consequences

- ПАРАЛЛЕЛЬ МЕЖДУ ГРАФОМ ЮНГА И ГРАФОМ СПЕКТРОВ.
- МЕРА ПЛАНШЕРЕЛЯ И ГАУСОВ АНСАМБЛЬ (GOE, GUE).

The concepts of continuous (continual) graded graphs of a special type are introduced. For such kind of continuous graphs the sets of finite paths are convex finite-dimensional compacts, and the central measures are defined by normalized Lebesgue measures on these compacts. They set co-transition probabilities of the central measures. The main example of such graphs is the Gelfand-Zetlin type graphs, and graph of spectra of infinite Hermitian matrices. The problem of describing central measures on the set of paths of such graphs acquires a new character in comparison with previous works on this topic (Pickrel, Vershik-Olshansky), and reduces to the establishment of surprising 0-1 laws for non-stationary Markov chains, or in another way to problems of coincidence or mismatch of geometric and general boundaries of random walks. There is an amazing internal parallelism between lists of central measures of degenerate type for Hermitian matrices (Wishart measures) and for Young graph (discrete Thoma measures) . Even more surprising is the internal similarity between the standard Gaussian measure (GOA or GUI) on matrices and the Plancherel measure on infinite Young diagrams.