Modeling of bumping routes in the RSK algorithm

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The Robinson-Schensted-Knuth algorithm defines a bijection between sets of sequences of a linearly ordered set and pairs of Young tableaux of the same shape: a semi-standard tableau (SSYT) P and a standard tableau (SYT) Q. At the same time, it is known [1] that RSK establishes the correspondence between a uniform measure on sequences and a Plancherel measure on the shapes of Young tableaux. A sequence of values bumped during a single iteration of the algorithm in the P tableau forms the so-called "bumping route". In Fig. 1 is an example of a bumping route in a tableau of 50 boxes when processing the number 18.

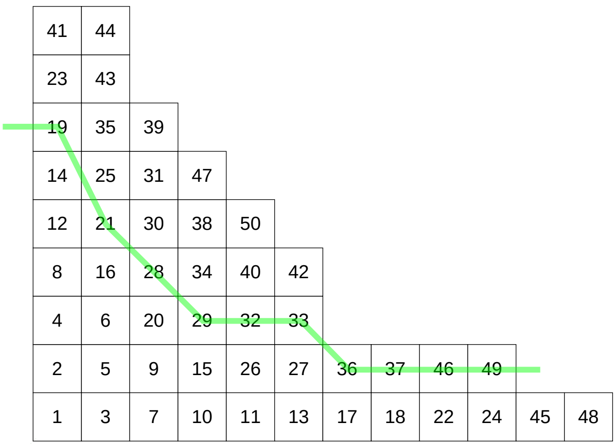

FIGURE 1. An example of a bumping route

The explicit formulae for the limit bumping routes in tableau P which correspond to uniformly-distributed random sequences were obtained in [2]. However, the question remains open about how the bumping routes converge to their limit curves with increasing size of Young tableaux.

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A series of computer experiments [3, 4] were conducted in order to investigate this problem. It was studied how the distance between the bumping routes and their limit shapes changes with the size of the tableaux. We considered SSYT filled with real numbers from the range [0, 1]. Tableaux sizes were taken from the range of $n \in [10^5, ..., \cdot 10^7]$ boxes with a step of 10^5. A fixed number of Young tableaux P of each of the considered sizes was generated. Then, we inserted various input values of α in the resulting tableaux and calculated the averages and variances of the deviations of the discrete bumping routes from the corresponding limit curves. The computer experiments show that the distance between the bumping routes and the limit shapes is well approximated by the formula

$$f(n) = a \cdot n^{-\frac{1}{4}} + b \cdot n^{-\frac{1}{2}}.$$

Note that the first term of this equation shows the extremely slow convergence of the bumping routes to their limit shapes. Fig. 2 shows the average distances of the bumping routes from the limit shapes for various input values α.

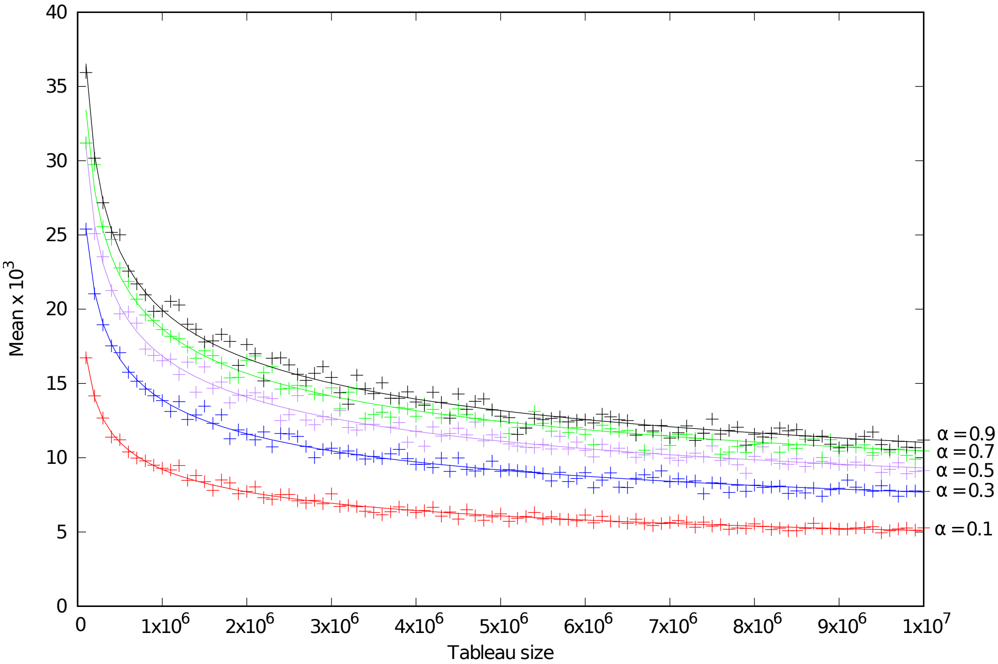

FIGURE 2. Mean values of deviations of bumping routes from their limit shapes and corresponding approximating curves

Also, as a result of experiments, it was found out that the distribution of the ends of the bumping routes at the profile of Young tableaux is quite close to Gaussian [4]. We estimated the parameters of the Gaussian distribution depending on the values $\alpha = 0.1, 0.3, 0.5, 0.7, 0.9$ fed to the input of the RSK algorithm.

References

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