Integration of new mathematical ideas into engineering curriculum: case of tropical mathematics

Victoria Kazakevich, Sergey Kolonitskii and Elena Tolkacheva

Abstract. The case of introduction of Tropical Mathematics at the Faculty of Computer Science and Technology, St. Petersburg Electrotechnical University "LETI", is considered. Using it as an example, a tentative scheme of introduction of a new area of research to the existing education process is presented.

Introduction

Most results taught in basic mathematical courses at engineering higher education establishments (such as algebra or calculus) date back to XVIII – XIX centuries. In this aspect the discrete line of courses developed at LETI ("Discrete Mathematics and Computer Science", "Combinatorics and Graph Theory", "Mathematical Logic and Theory of Algorithms") are a refreshing exception, as many results discussed in these courses date to the second half of XX century or even to XXI century. But acquainting students with current state of mathematics and its applications which are most relevant to the students remains a burning question. In order to develop applied areas and introduce new ideas into engineering practice as soon as possible, it is important to introduce familiarisation with modern areas of research into the educational process. Such introduction is beset with a variety of obstacles, beginning with a rather limited available time and ending with a certain conservatism of the system of education. In 2019 the deptartment of Algorithmic Mathematics initiated the development of pegagogical technologies of introduction of new mathematical ideas into general education process. A new area of research must satisfy a set of criteria to be eligible for such an introduction:

1. Recently emerged areas have an advantage: they usually have a more moderate learning curve, and there probably are easy unsolved problems.
2. The area must be relevant for our students in their professional capacity, i.e. it must have enough applications, the relevance of which should be obvious

(and ideally should be heard ofeven by non-specialists) and easily confirmed by examples. This also alleviates the problem of motivating the students.

3. The area should be immersed in context the students are in, both educationally and professionally. This means that the area should be grounded in what our students learned before, it should tie in with what they are learning at the moment, and finally it should be applicable to their professional area of expertise. Ideally such area of research should provide a whole spectrum of opportunities, ranging from purely applied use in the industry for those who will use it only as one of possible tools of profession, to the opportunity to actually pursue research in it.
4. The area should have researchers willing to cooperate with the university. Introduction of a new area means it is necessary to grow and educate a cadre of teachers first. We propose that collaboration with leading researchers can be a successful center of crystallization of such a process. Of course, using leading researchers to teach fresh students is a waste of resources, but they can and should help with source material, read an introductory course for teachers, oversee seminars and act as scientific advisor for whose who want to pursue research, both students and teachers.

Tropical mathematics fit these criteria ideally. The area is young (Wikipedia dates its emergence to early 2000's), although many relevant ideas percolated long before it. The learning curve is not too steep, especially for students that paid attention at algebra and algebraic structures. Just saying "neural nets" is enough to establish its relevance. It also helps to bring the students into the context, as does the emergence of familiar terms, starting with the most basic, such as "semiring" or "piecewise-linear function". Students who have already covered graph theory can use the problem of finding the shortest paths in a graph as a relevant and manageable example, as it has a very compact and elegant interpretation in tropical terms. And last but not the least, D. Y. Grigoriev and N. N. Vasiliev have agreed to collaborate with us. Thus was made the decision that tropical mathematics was coming to LETI. By this moment the department also had members willing to pursue reseacrh in this area.

1. Stages of integration

The experimental integration of tropical mathematics into LETI's educational process was planned as a sequence of stages.

1.1. Stage 0

D. Y. Grigoriev read a small series of open lectures on tropical mathematics at LETI. The lectures proved to be of interest both to students and the faculty. Objectives at this point were:

- Familiarize with the term "tropical mathematics";
- Gauge the students´ response to the presented area;

- Notify the students that faculty is interested in such activities;
- Show the students — albeit rather superficially — what modern mathematical research looks like.

Results of stage 0 were:

- Recorded lectures of D. Y. Grigoriev, which were made available and which are, if views are any indicator, in demanhd;
- Quite substantial discussion during the lectures;
- Emergence of a group of interested students, which comprised the core of one branch of experiment during stage 1.

1.2. Stage 1

Stage 1 was comprised of three branches developing in parallel:

1. Student seminar on tropical mathematics and neural nets. This seminar alternated reports on tropical mathematics and neural nets. Articles on tropical mathematics were supplied by D. Y. Grigoriev. Faculty members also attended this seminar, acting as audience and consultants. Seminar persisted in this form for several years.
2. Seminar on algorithmic mathematics, supervised by N. N. Vasiliev, with mixed audience including students, graduate students and faculty. Some reports were on tropical mathematics. Seminar persists to this day.
3. D. Y. Grigorievś and N. N. Vasilievś collaboration with departmentś members, master students and graduate students. This collaboration is ongoing.

Results of stage 1 were:

- Familiarisation with the term "tropical mathematics";
- Students learned enough to discuss tropics themselves, and acquired a habit of doing so. Simultaneously, an environment formed which encouraged such discussion, analysis and idea exchange;
- Emergence of a group of senior students who could supervise or consult junior students;
- Emergence of department members who could answer questions on the topic of tropical mathematics or act as an advisor for a work over specific problem, an alternative exam, or even a barchelorś or masterś thesis. It is important to stress that these department members were known as such to the students.

1.3. Stage 2

During the second stage (which is currently underway) a regular schedule of seminar work was established. Student seminars in small groups (up to 15 members) are regularly conducted, with authors as supervisors. For 4 years this seminar was conducted as an elective course for senior year barchelor students. This year the seminar also took place for second-year master students of our department (it should be noted that it was the first batch of master students our department had). Beside that, each year several freshmen take up tropical mathematics as a topic for alternative exam. Usually they are supervised by senior students, but one

time this group was too big, and authors had to step in as supervisors. Results of stage 2 are:

- Tropical mathematics has firmly secured a place in the curriculum, both formally and psychologically.
- Neural nets as the main application considered completely solve the problem of motivation and relevance.
- Number of students acquainted with tropical mathematics has grown considerably. The environment that was formed during the previous stage has also grown. For example, the students´ university "IT LETI" (a departments spin-off project) has a course on tropical mathematics.
- Methodics of teaching tropical mathematics to students of LETI are in the process of development.

Conclusion

All above is a work already done. Plans for the next stage include scaling the teaching of tropical mathematics both in the size of student group and the amount of alotted time. Tropical mathematics are slated to be offered as an elective for all senior-year barchelor students of FCST. "Basics of tropical mathematics" course is planned to be introduced for barchelors, while our department opens its own barchelor program. A number of difficulties is expected to arise as a consequence of such a scaling. Ways of mitigating them are being developed.

Victoria Kazakevich
Dept. of Algorithmic Mathematics
Saint Petersburg Electrotechnical University "LETI"
Saint-Petersburg, Russia
e-mail: sokratt@gmail.com

Sergey Kolonitskii
Dept. of Algorithmic Mathematics
Saint Petersburg Electrotechnical University "LETI"
Saint-Petersburg, Russia
e-mail: sergey.kolonitskii@gmail.com

Elena Tolkacheva
Dept. of Algorithmic Mathematics
Saint Petersburg Electrotechnical University "LETI"
Saint-Petersburg, Russia
e-mail: eatolkacheva@etu.ru