Implementation of A.N. Krylov series convergence acceleration in the CAS Sage

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Abstract. The problem of calculating the sum of the Fourier series in computer algebra systems is considered. The work presents some functions of the «Kryloff for Sage» software package. This package is designed to accelerate the convergence of Fourier series for problems of mathematical physics. In some cases, it is possible to obtain an expression in finite terms.

The solutions of the problems of mathematical physics are usually represented in the form of Fourier series. At the same time, many initial-boundary value problems for the wave equation can be solved in finite terms. An attempt to get a presentation in finite terms using standard series summation functions built into computer algebra systems leads to some difficulties, see [1]. The expressions for sums obtained in computer algebra systems contain special functions and numerous branches. These difficulties are related to the fact that the Fourier series for the wave equation are not the analytical functions of their arguments. Therefore they are not elementary functions in the sense of Liouville theory.

To solve this problem, it is proposed to consider not the task of summing up the series in finite terms form, but the task of accelerating its convergence. We will use the method of accelerating the convergence systematically described in A.N. Krylov works. As Krylov noted, this method «often leads to the representation of the sum of the proposed series inclosed form under the guise piecewice function» [2]. For example, this is true for several Green's functions of the wave equation on the segment [3]. The guise piecewice function will not be elementary in the sense of Liouville theory. However, it will be elementary function in the modern sense of this concept [4].

In this work the first functions of the software package «Kryloff for Sage» in CAS Sage [5] are presented. We consider a problem when the function represented by a given Fourier series is not known. We are implemented in Sage the variants of this method, which can lead to the definition of the desired function in finite terms [6, 7].

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