Painleve Test and Integrability of Polynomial ODEs

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Abstract. We calculate Painleve Test of integrability for some integrable polynomial ODEs. Some of them don't pass Painleve Test.

Introduction

The essence of Painleve Test of integrability polynomial ODEs is solving this equations in form of Puiseux series with finite nonzero principal part [1]. We calculate that series for solutions of following integrable ODEs: Lotka-Volterra system [2], Chazy equation [3], Euler-Poisson system ODEs described rigid body motion around a fixed point [4].

Results

For calculations of that Puiseux series we used algorithms and programs described in [5, 6]. Solutions of Lotka-Volterra system and Chazy equation don't have that Puiseux expansions and don't pass Painleve Test. For solutions of Euler-Poisson system that Puiseux expansions exist under certain conditions on the parameters of the system. The set of calculated conditions include all known contitions of Euler-Poisson system integrability. Other calculated conditions point to new integrability case possibility.

Conclusion

If polynomial ODEs dont't pass Painleve Test, then first integrals are complicated than algebraic integrals.

References

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