The software GInv for calculating involutive bases of polynomial ideals

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Abstract. A new version of the softwarefor finding the involutive ideals of polynomial and differential ideals is presented, it called GInv. A brief description of the improvements is given and the results of its testing on real problems are presented, including the problem about the nodes and weights of cubature formulas on the sphere.

Introduction

The study of problems in mechanics and mathematical physics is often reduced to solving systems of polynomial equations. However, as rightly noted by the authors of Numerical recipes [1], this problem does not admit a universal numerical solution method in the cases of two and more unknowns. Fortunately, in the 1990s, an implementation of the Buchberger's algorithm for finding Gröbner bases of ideals of polynomial rings appeared in computer algebra systems (SKA). Finding the Gröbner basis in the lexicographic order of monomes allows us to reduce the solution of a system of nonlinear equations with a finite number of solutions to the solution of one equation with one unknown [2]. Although the Buchberger's algorithm allows to find the Gröbner basis in a finite number of steps, in practice on a modern computer it can be used to solve systems of a small degree and with no more than a dozen unknowns. Several improvements to this algorithm were proposed in the 1990s, some of which remain commercial. At the same time, Gerdt, Zharkov and Blinkov [3] proposed a new approach to finding the bases of polynomial ideals based on the original concept of involutive division. In the 2000s, this algorithm was implemented in the form of GInv software and applied to the study of a number of problems in mathematical physics [4]. Recently, this software has been completely redesigned and transferred to the public domain [5].

1. New version

The new version of GInv is written as a library in C++11. For dynamic data structures such as lists, red-black and binary trees, GMP libraries are used. This allows to complete calculations with arbitrary precision integers and object-oriented memory redistribution. The package interface is designed as an additional module, written in Python3 language. Attention was paid to optimizing the use of memory and processor cache. At the same time, the well-known disadvantages of the standard malloc/free and the garbage collection approach were taken into account. The original approach turned out to be easy to implement compared to standard malloc/free and convenient when searching for errors related to memory leaks.

Improving the methods of calculating the Gröbner basis has two goals: speeding up calculations and reducing the resources used in calculations. These goals are not always possible to combine. From general considerations, it is obvious that very frequent garbage collection can slow down calculations, and on the contrary, the willingness to provide the system with an unlimited number of clean memory pages can lead to a very rapid exhaustion of resources. Moreover, it is not always clear whether a particular change will lead to an actual improvement, that is, the achievement of one of these goals. Therefore, it is extremely important to conduct systematic testing at every stage of the development of such systems. The new version of GInv was tested on a real problem – the problem of calculating nodes and weights of cubature formulas on a sphere cite [6]. This problem leads to a very complex system of nonlinear equations, which previously could only be solved numerically. The economical attitude to memory made it possible to solve it analytically.

2. Testing

A special software tool has been developed that allows to test various versions of GInv and similar computer algebra systems in automatic mode and get the results in graphical form. To proceed with testing a fairly large and representative set of 135 equations was assembled. This test database was updated to use JSON format, suitable for use in various computer algebra systems (GInv, SymPy, Sage).

This article will describe this tool and present the results of testing the current version of GInv. The source files of the testing are publicly available and available for download in a separate GitHub repository at [7].

The system was tested on a server platform consisting of two 4-core Intel Xeon L5630 processors. Each processor had 4 computing cores with support for Hyper-Threading technology, which allowed running 2 threads on each physical core. Thus, the total number of logical cores (processing threads) was 8. The base clock frequency of each processor core was 2134 MHz. Some results of this testing are displayed on following table 1. Full terults available at [7].

Test	Dimension	Length of the Basis	Reduction	Time
ilias13	7	1	0	0,53
comb3000	10	35	503	11,70
hcyclic6	7	221	18634	308,41
eco9	9	189	159992	4322,05
hcyclic7	8	1182	542213	56018,99

TABLE 1. Sample of test results

Conclusion

A new version of software GInv was developed and tested. This new version and dataset for testing are now publicly available on open sources. The new version allowed to solve some problems analytically, which were previously solvable only numerically.

References

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