The Euler top and the Lagrange top as two special cases of the Galois top

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The MacCullagh ellipsoid of inertia is transformed if we relocate its centre along the Galois axis [7, 8, 9, 11, 13, 14, 15]. The two principal axes, corresponding to extreme moments of inertia would rotate (with respect to rigid body reference frame), whereas the direction of the principal axis, corresponding to the intermediate moment of inertia is preserved (remaining orthogonal to the Galois axis). The latter (newly emerging, rotated and rescaled)[1] MacCullagh ellipsoid of inertia shares the same Galois axis with the former ellipsoid. We have thus ensured that the Galois top, that is, a heavy top in a uniform gravitational field which fixed point lies at a Galois axis, is well-defined.

The Euler top and the Lagrange top are thereby seen as two special cases of the Galois top. The Euler top corresponds to the special case of the Galois top which fixed point coincides with its centre of mass, whereas the Lagrange top corresponds the special case of the Galois top which Galois axis coincides with a principal axis of inertia, that is, the case where two principal moments of inertia did coincide one with the other.[2]

The Galois essential and alternative elliptic functions, which were discussed in [1, 2, 3, 4, 5, 6, 10, 12, 14], provide the natural means for constructing explicit solutions to both Euler top and Lagrange top, as well as, vividly exhibiting their symmetries.

References

[1] Adlaj S. *Mechanical interpretation of negative and imaginary tension of a tether in a linear parallel force field.* Selected Works of the International Scientific Conference on Mechanics "Sixth Polyakhov Readings", Saint-Petersburg (2012): 13–18.

[1]Recall that the length of a principal axis of MacCullagh ellipsoid is proportional to the square root of the corresponding principal moment of inertia.

[2]In other words, the Lagrange top corresponds the special case of the Galois top which (both) Galois axes merge in a single axis.

[2] Adlaj S. *An analytic unifying formula of oscillatory and rotary motion of a simple pendulum* (dedicated to the 70th birthday of Jan Jerzy Slawianowski, written in 2013 and updated in 2014). Proceedings of International Conference "Geometry, Integrability, Mechanics and Quantization", Varna, Bulgaria, 2014, June 6-11. Printed by "Avangard Prima", Sofia, Bulgaria, 2015: 160–171. Available at https://semjonadlaj.com/SP/JJS70.pdf.

[3] Adlaj S. *On the second memoir of Évariste Galois' last letter.* Computer Tools in Education, 2018 (4): 11–26. Available at https://doi.org/10.32603/2071-2340-4-11-26.

[4] Adlaj S. *Elliptic integrals, functions, curves and polynomials.* Computer Assisted Mathematics, 2019 (1): 3–8. Available at http://cte.eltech.ru/ojs/index.php/cam/article/view/1626/1604.

[5] Adlaj S. *Galois elliptic function and its symmetries.* Polynomial Computer Algebra International Conference, St. Petersburg, Russia, 2019, April 15-20: 11–17. Available at https://pca-pdmi.ru/2019/files/37/PCA2019SA.pdf.

[6] Adlaj S. *Multiplication and division on elliptic curves, torsion points and roots of modular equations.* Zap. Nauchn. Sem. POMI, 2019 (485): 24–57. Available at https://www.mathnet.ru/links/f2671399de97e3be75b38f5b0ace5adc/znsl6868.pdf.

[7] Adlaj S. *Torque free motion of a rigid body: from Feynman wobbling plate to Dzhanibekov flipping wingnut.* Available at https://semjonadlaj.com/SP/TFRBM.pdf.

[8] Adlaj S. *Galois axis.* International Scientific Conference "Infinite-Dimensional Analysis and Mathematical Physics" (Dedicated to the memory of Sergei Vasilyevich Fomin), Moscow, Russsia, January 28 - February 1, 2019: 9–11. Available at https://semjonadlaj.com/Galois/GaloisAxis190129.pdf.

[9] Bulyzhenkov I. *Vertexes in kinetic space-matter with local stresses instead of localized particles with distant gravitation.* DOI:10.20944/preprints202102.0477.v1.

[10] Lamarche F. & Ruhland H. *Computed multivalues of AGM reveal periodicities of inverse functions.* Computer tools in education, 2022 (3): 64–81. Available at http://cte.eltech.ru/ojs/index.php/kio/article/view/1758.

[11] Seliverstov A. *On circular sections of a second-order surface.* Computer tools in education, 2020(4): 59–68. DOI: 10.32603/2071-2340-2020-4-59-68. Available at http://ipo.spb.ru/journal/index.php?article/2258/.

[12] Адлай С.Ф. *Равновесие нити в линейном параллельном поле сил.* Saarbrucken, LAP Lambert Academic Publishing, 2018, 88 pages. ISBN 3659535427, 978-3-659-53542-0.

[13] Всеобщий словарь ремёсел и наук. *Ось Галуа.* Статья доступна по ссылке https://cyclowiki.org/wiki/%D0%9E%D1%81%D1%8C_%D0%93%D0%B0%D0%BB%D1%83%D0%B0.

[14] Ламарше Ф. & Адлай С.Ф. *Комплексные периоды, обратимость по времени и двойственность в классической механике.* Заседание Российского междисциплинарного семинара по темпорологии им. А.П. Левича 26 ноября 2019 г. Выступление и его обсуждение доступно по ссылке https://www.youtube.com/watch?v=8DN5vQOP5Cw.

[15] Мисюра Н.Е. & Митюшов Е.А. *Кватернионные модели в кинематике и динамике твердого тела.* Издательство: Екатеринбург: Уральский федеральный университет, 2020, 120 стр. ISBN 978-5-7996-3150-5.

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