Algorithmic mathematics in a technical university: different ways to comprehend the material

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Abstract. The problem of a student's comprehension of mathematical concepts is becoming increasingly relevant in modern conditions, when mental activity to comprehend the material is often replaced on the student's initiative by searching for ready-made answers on the Internet or using symbolic algebra systems . The report presents a number of pedagogical experiments aimed at clarifying the conditions and methods of supporting the processes of comprehending new material in a rich external information environment.

Introduction

Bourbaki's traditions [1] in the presentation of mathematics influenced the teaching of mathematics in technical universities. Implicitly, and sometimes explicitly, the idea was postulated that formal mathematical structures correspond to analogous intellectual structures in the learner's brain. From which the conclusion was drawn: if abstract mathematical structures are strictly consistently presented, moving from the general to the particular, then in the student's head there will be a structure of concepts and mental operations that is adequate to Bourbaki's books. However, the process of acquiring knowledge is much more complex and formal structures do not become a thinking tool if they are not based on already existing mental structures. Marvin Minsky [2] called this the investment principle. In other words, understanding the material is not so much a function of correctly organized mathematical material, but rather a function of all the experience accumulated up to a given moment.

It can be assumed that the effect of understanding is associated with prediction (the development of the idea of the area of proximal development [3]): even before the teacher finishes the sentence, the listener already has some model in his head of what the teacher wants to convey to the student. Two options are possible: either this model does not contradict what the teacher says and noninsight comprehension occurs [4], or a contradiction between the model and a new

concept arises and then either insight occurs, that is, a new gestalt arises [5] - a new look at the material being presented (instant restructuring information [6]), or the contradiction persists for a longer period.This contradiction is a mechanism of delayed comprehension.

1. Summary of the report

The report discusses the following ways to understand new mathematical concepts:

1) The traditional way of conveying meaning is activity-based. In the process of its implementation, the teacher forms in the student pre-set skills that form a contextual environment for operating objects in the subject area. This specially formed contextual information environment is associated with semantic constructions that are objectified in the context of this environment.

2) Construction of algorithms that implement constructive descriptions of the properties of mathematical objects.

3) Setting tasks and interacting with systems that verify hypotheses using many examples of the subject area.

The report also describes an experiment in which the features of spontaneous concept formation were studied based on the subconscious construction of predictive models.

Conclusion

For future engineers, mathematics is important as a tool, but if earlier Krylov's words [7] that mathematics for an engineer is a tool "like an ax and a saw for a carpenter" could be interpreted as the presence of a set of applied skills, now the presence of intellectual tools changes the meaning of Krylov's statement. Now the "Krylov's tool" becomes an intellectual toolkit based on the understanding of abstract mathematical concepts. To achieve this goal, it is necessary to replace the formally mathematical approach a la Bourbaki with an "investment" approach based on the existing knowledge, ideas and experience of students.

References

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